Day 32 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that its position vector relative to O at time t s is $\mathbf{r} = (3t^2 + 3t)\mathbf{i} + (t^3 + t)\mathbf{j}$ metres.
Find the velocity when t = 3, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (6t + 3)\mathbf{i} + (3t^2 + 1)\mathbf{j}$
$t = 3$: $\mathbf{v} = (21)\mathbf{i} + (28)\mathbf{j}$
Speed $= \sqrt{(21^2 + 28^2)}$
Answer: $\mathbf{v} = (21)\mathbf{i} + (28)\mathbf{j}$ m s^{-1}; speed 35 m s^{-1}

2 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + 7n + 13$ is odd for all integers n.

$n^2 + 7n = n(n + 7)$
As 7 is odd, n and n + 7 have opposite parity,
so one of them is even and the product n(n + 7) is even.
Even + 13 = odd, so $n^2 + 7n + 13$ is odd.
Answer: n(n + 7) is even, so the expression is odd

3 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$$\int_0^1 \frac{6x + 3}{x^2 + x + 1}\,dx,$$

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator
$\int \frac{6x + 3}{x^2 + x + 1}\,dx = 3\ln(x^2 + x + 1)$
$= [3\ln(x^2 + x + 1)]_0^1$
$= \ln 27$
Answer: ln 27

4 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + 3x^2 + kx$.
Given that (x + 4) is a factor of f(x),
(a) find the value of k,
(b) factorise f(x) completely.

$f(-4) = 0$: $-64 + 48 - 4k = 0$
$k = -4$
$f(x) = (x + 4)(x^2 - x)$
$= (x - 1)x(x + 4)$
Answer: (a) k = −4 (b) f(x) = (x − 1)x(x + 4)

5 Grouped data: mean and median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises 46 measurements of t.
(a) Estimate the mean.
(b) Estimate the median using linear interpolation.

(a) Using mid-points: $\Sigma fx = 1400$, so mean $= 1400 \div 46 = 30.4$

(b) $\frac{n}{2} = 23$. Cumulative frequency before the median class = 21

Median class 30 to 35. Median $= 30 + \frac{2}{11} \times 5 = 30.9$

Answer: mean ≈ 30.4, median ≈ 30.9

6 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 1 and sum to infinity 2.
Find the common ratio and the value of the 3rd term.

$S_\infty = \frac{a}{1-r}: 2 = \frac{1}{1-r}$

$1 - r = \frac{1}{2}$, so $r = \frac{1}{2}$

$u_3 = ar^2 = 1 \times \frac{1}{2}^2 = \frac{1}{4}$

Answer: $r = \frac{1}{2}$, $u_3 = \frac{1}{4}$

7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{1}{(x+2)(x+3)}$ in partial fractions.

(b) Hence evaluate $\int_0^2 \frac{1}{(x+2)(x+3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{1}{(x+2)(x+3)} \equiv \frac{A}{x+2} + \frac{B}{x+3}$, so $1 \equiv A(x+3) + B(x+2)$
$x = -2$: $A = 1$; $x = -3$: $B = -1$

(b) $\int_0^2 \left(\frac{1}{x+2} + \frac{-1}{x+3}\right) dx = [\ln(x+2) - \ln(x+3)]_0^2$
$= \ln\frac{4}{2} - \ln\frac{5}{3} = \ln\left(\frac{6}{5}\right)$

Answer: (a) $A = 1$, $B = -1$ (b) $\ln\left(\frac{6}{5}\right)$

8 Work-energy: speed after falling or sliding (CIE)

A bead of mass 5 kg is released from rest 10 m above the ground and falls freely.
Take $g = 9.8$ m s^{-2}.
Use energy to find its speed on reaching the ground.

Loss of PE = gain of KE: $m \times 9.8 \times 10 = \frac{1}{2} m v^2$
$v^2 = 19.6 \times 10 = 196$
$v = 14$
Answer: 14 m s^{-1}

9 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = e^x$, the x-axis
and the lines $x = 0$ and $x = \ln 2$.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^{\ln 2} y^2\,dx = \pi \int_0^{\ln 2} e^{2x}\,dx$

$= \pi\left[\frac{e^{2x}}{2}\right]_0^{\ln 2}$

$= \frac{3\pi}{2}$

Answer: $\frac{3\pi}{2}$

10 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 52) = 0.1587 and P(X > 76) = 0.0228.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.999$

$52 = \mu + (-0.9998)\sigma$ and $76 = \mu + 1.999\sigma$

Subtract: $24 = 2.999\,\sigma$, so $\sigma = 8.00$

$\mu = 52 - (-0.9998)(8.003) = 60.0$

Answer: $\mu = 60.0$, $\sigma = 8.00$