Day 319 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A ship moves with constant velocity $(-2\mathbf{i} - 3\mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(4\mathbf{i} + 6\mathbf{j})$ m.
Find (a) its position vector when t = 4, (b) the value of t when it crosses the line through O in the direction of $\mathbf{i}$.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (4\mathbf{i} + 6\mathbf{j}) + 4(-2\mathbf{i} - 3\mathbf{j}) = (-4\mathbf{i} - 6\mathbf{j})$
(b) $\mathbf{j}$ component zero: $6 - 3t = 0$, so $t = 2$
Answer: (a) $(-4\mathbf{i} - 6\mathbf{j})$ m (b) t = 2 s

2 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + 2x^2 + kx - 6$.
Given that $(x + 3)$ is a factor of f(x),
(a) find the value of k,
(b) factorise f(x) completely.

$f(-3) = 0$: $-27 + 18 - 3k - 6 = 0$
$k = -5$
$f(x) = (x + 3)(x^2 - x - 2)$
$= (x - 2)(x + 1)(x + 3)$
Answer: (a) k = −5 (b) f(x) = (x − 2)(x + 1)(x + 3)

3 Inverse normal (All boards)

The length X cm of a metal rod is normal with mean 40, s.d. 6.
Find k (3 s.f.): (a) P(X < k) = 0.05,
(b) P(X > k) = 0.01.

(a) $z = -1.645$, so $k = 40 + (-1.645) \times 6 = 30.1$
(b) P(X < k) = 0.99, $z = 2.326$, so $k = 40 + 2.326 \times 6 = 54.0$
Answer: (a) 30.1 (b) 54.0

4 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards with speed 26 m s^{-1} from ground level.
Take g = 9.8 m s^{-2}. Find, to 3 significant figures,
(a) the greatest height reached, (b) the time of flight.

(a) $0 = 26^2 - 2 \times 9.8 \times H$, so $H = 676 \div 19.6$
(b) s = 0: $t = 2 \times 26 \div 9.8$
Answer: (a) 34.5 m (b) 5.31 s

5 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) y = f(3x)
(b) y = f(−x)
(c) y = −f(x)

(a) y = f(3x) is a stretch, scale factor $\frac{1}{3}$, parallel to the x-axis.
(b) y = f(−x) is a reflection in the y-axis.
(c) y = −f(x) is a reflection in the x-axis.
Answer: (a) stretch, scale factor $\frac{1}{3}$, parallel to the x-axis (b) reflection in the y-axis (c) reflection in the x-axis

Day 319 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ 5 \\ -2 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a}\cdot\mathbf{b} = (0)(5) + (-2)(5) + (1)(-2) = -12$

$|\mathbf{a}| = \sqrt{5}$, $|\mathbf{b}| = \sqrt{54}$

$\cos\theta = \dfrac{-12}{\sqrt{5}\sqrt{54}} = -0.7303$

$\theta = 136.9°$

Answer: 136.9°

7 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.3. It is tossed 20 times and lands heads 2 times.
Test, at the 5% significance level, whether the probability of heads is different from 0.3.

H_0: $p = 0.3$, H_1: $p \neq 0.3$. Under H_0, $X \sim B(20, 0.3)$

$P(X \leq 2) = 0.0355$

Two-tailed: compare with 0.025. $0.0355 > 0.025$, so do not reject H_0

There is insufficient evidence that the probability of heads is different from 0.3

Answer: do not reject H_0: insufficient evidence that the probability of heads is different from 0.3

8 Rates of change (connected rates) (All boards)

The area of a square increases at a constant rate of 6 cm^2s^{-1}.
Find the rate of increase of the side length when the side is 4 cm.

$A = x^2$, so $\dfrac{dA}{dt} = 2x\dfrac{dx}{dt}$

$6 = 2 \times 4 \times \dfrac{dx}{dt}$

$\dfrac{dx}{dt} = \dfrac{3}{4}$

Answer: $\dfrac{3}{4}$ cm/s

9 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $5x - 9y = 35$ and line l_2 has equation $5x + 3y = -25$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (−2, −5)

l_1 meets $y = 0$ at $x = 7$, and l_2 meets $y = 0$ at $x = -5$

Base = 12, height = 5

Area = $\dfrac{1}{2}$ × base × height = 30

Answer: 30

10 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $2x^2y + 3y^3 - xy = -99$.

Find $\dfrac{dy}{dx}$ in terms of x and y, and the gradient at (2, −3).

Differentiate with respect to x: $2x^2\dfrac{dy}{dx} + 4xy + 9y^2\dfrac{dy}{dx} - x\dfrac{dy}{dx} - y = 0$

$\dfrac{dy}{dx} = \dfrac{y(1 - 4x)}{2x^2 + 9y^2 - x}$

At (2, −3): gradient = $\dfrac{7}{29}$

Answer: $\dfrac{dy}{dx} = \dfrac{y(1 - 4x)}{2x^2 + 9y^2 - x}$; gradient $\dfrac{7}{29}$