Day 318 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 5 s, (b) the distance between the stations.

(a) a = gradient = 20 ÷ 5
(b) Distance = area of trapezium = ½ × (41 + 30) × 20
Answer: (a) 4 m s^{-2} (b) 710 m

2 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 3 cm and angle 45°.
Find, in terms of π, the arc length and the area of the sector.

$\theta = 45° = \frac{\pi}{4}$ radians

Arc length = $r\theta = \frac{3\pi}{4}$ cm

Area = $\frac{1}{2}r^2\theta = \frac{9\pi}{8}$ cm^2

Answer: arc $\frac{3\pi}{4}$ cm, area $\frac{9\pi}{8}$ cm^2

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(4x)^3 \times 4x^2}{8x^2}$.

$(4x)^3 = 64x^3$
Numerator = $64 \times 4x^5 = 256x^5$
Divide: $\frac{256}{8} = 32$ and $x^5 \div x^2 = x^3$
Answer: $32x^3$

4 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 48 N in the negative x-direction and a force of Q N in the negative y-direction.
$\cos\theta = \frac{12}{13}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 48$, so $P \times \frac{12}{13} = 48$
P = 52
Resolve vertically: $Q = P\sin\theta = 52 \times \frac{5}{13}$
Answer: P = 52 N; Q = 20 N

5 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:
(a) y = f(−x) (b) y = f(x) − 1

(a) y = f(−x): A(−2, −5) maps to (2, −5), B(2, 1) maps to (−2, 1)
(b) y = f(x) − 1: A(−2, −5) maps to (−2, −6), B(2, 1) maps to (2, 0)
Answer: (a) (2, −5), (−2, 1) (b) (−2, −6), (2, 0)

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6 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{6}{(x+2)(x+5)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{6}{(x+2)(x+5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{6}{(x+2)(x+5)} \equiv \frac{A}{x+2} + \frac{B}{x+5}$, so $6 \equiv A(x+5) + B(x+2)$
$x = -2$: $A = 2$; $x = -5$: $B = -2$

(b) $\int_0^1 (\frac{2}{x+2} + \frac{-2}{x+5})\,dx = [2\ln(x+2) - 2\ln(x+5)]_0^1$
$= 2\ln\frac{3}{2} - 2\ln\frac{6}{5} = \ln(\frac{25}{16})$

Answer: (a) A = 2, B = −2 (b) $\ln(\frac{25}{16})$

7 Finding the mean or standard deviation (All boards)

A battery lasts X hours, normal with mean 250.
Given P(X > 215.1) = 0.99, find the standard deviation (3 s.f.).

P(X < 215.1) = 0.01, so $z = -2.326$
$\frac{215.1 - 250}{\sigma} = -2.326$
$\sigma = -34.9 \div -2.326 = 15.0$
Answer: σ = 15.0

8 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $24\cos\theta + 7\sin\theta \equiv 25\cos(\theta - 16.3°)$,
solve $24\cos\theta + 7\sin\theta = -3$ for $0 \le \theta \le 360°$. (1 d.p.)

$25\cos(\theta - 16.3°) = -3$, so $\cos(\theta - 16.3°) = \frac{-3}{25}$
$\theta - 16.3° = \pm 96.9°$
$\theta = 16.3° + 96.9° = 113.2°$
$\theta = 16.3° - 96.9° = -80.6°$, so $\theta = 279.4°$
Answer: θ = 113.2°, 279.4°

9 Binomial two-tailed test (All boards)

A factory says a proportion 0.25 of its parts are defective. In a sample of 50 parts, 6 are defective.
Test, at the 5% significance level, whether the proportion is different from 0.25.

H_0: $p = 0.25$, H_1: $p \neq 0.25$. Under H_0, $X \sim B(50, 0.25)$
$P(X \le 6) = 0.0194$
Two-tailed: compare with 0.025. $0.0194 < 0.025$, so reject H_0
There is sufficient evidence that the proportion is different from 0.25
Answer: reject H_0: sufficient evidence that the proportion is different from 0.25

10 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sum_{r=1}^{n}(3r - 2) = 176$, find the value of n.

Sum $= \frac{n}{2}(1 + 3n - 2) = 176$
$3n^2 - n - 352 = 0$
$(n - 11)(3n + 32) = 0$
n must be positive, so n = 11
Answer: n = 11