Day 317 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Solving exponential equations using logarithms (All boards)

Solve $3^x = 2^{x+2}$, giving your answer to 3 s.f.

$x \ln 3 = (x + 2) \ln 2$
$x(\ln 3 - \ln 2) = 2 \ln 2$
$x = \dfrac{2 \ln 2}{\ln 3 - \ln 2} = 3.419$
Answer: x = 3.42

2 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x - 12$ meets the circle $x^2 + (y + 3)^2 = 65$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -x - 12$: $x^2 + (-x - 9)^2 = 65$
$x^2 + 9x + 8 = 0$
$(x + 1)(x + 8) = 0$, so $x = -8$ or $x = -1$
$y = -x - 12$: A = $(-8, -4)$, B = $(-1, -11)$
$AB^2 = 7^2 + (-7)^2 = 98$
Answer: A(−8, −4), B(−1, −11), AB = $7\sqrt{2}$

3 Trig equations in radians (All boards)

Solve $\tan x = 2$ for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

$\tan \theta = 2$ for $0 \leq x \leq 2\pi$
Calculator: x = 1.107, then use the symmetry of the graph
x = 1.107, 4.249
Answer: x = 1.11, 4.25

4 Inverse normal (All boards)

The length X cm of a metal rod is normal with mean 30, s.d. 5.
Find k (3 s.f.): (a) P(X < k) = 0.1,
(b) P(X > k) = 0.05.

(a) $z = -1.282$, so $k = 30 + (-1.282) \times 5 = 23.6$
(b) P(X < k) = 0.95, $z = 1.645$, so $k = 30 + 1.645 \times 5 = 38.2$
Answer: (a) 23.6 (b) 38.2

5 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(2n + 5)^2 - (2n - 5)^2$ is a multiple of 40 for all integer values of n.

$(2n + 5)^2 = 4n^2 + 20n + 25$
$(2n - 5)^2 = 4n^2 - 20n + 25$
Difference = $40n = 40 \times n$
n is an integer, so this is a multiple of 40 for every integer n.
Answer: difference = 40n, a multiple of 40

6 Velocity from variable acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its acceleration is $4t + 3$ m s^{-2}.
When t = 0 the velocity is -3 m s^{-1}. Find the velocity when t = 4.

$v = \int (4t + 3)\, dt = 2t^2 + 3t + c$
$t = 0$, $v = -3$ so $c = -3$
$t = 4$: $v = 41$
Answer: 41 m s^{-1}

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7 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 3x)^{-2}$ up to the x^2 term.

(b) Use $x = 0.03$ to estimate $\frac{1}{1.09^2}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = -2$, $a = 3$:

$= 1 - 6x + 27x^2$

(b) $1 + 3 \times 0.03 = 1.09$, so substitute $x = 0.03$:

$1 - 6 \times 0.03 + 27 \times 0.03^2 = 0.84430$

Answer: (a) $1 - 6x + 27x^2 + \ldots$ (b) 0.84430

8 Two independent binomials (All boards)

X ~ B(11, 0.4) and Y ~ B(7, 0.25) are independent.
Find P(X = 4 and Y ≤ 2), to 3 significant figures.

P(X = 4) = 0.2365
P(Y ≤ 2) = 0.7564
Independent, so multiply: 0.2365 × 0.7564 = 0.179
Answer: 0.179

9 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $5 \sin^2 x - 2 \cos x - 2 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2 x = 1 - \cos^2 x$: $3 - 5 \cos^2 x - 2 \cos x = 0$, i.e. $5 \cos^2 x + 2 \cos x - 3 = 0$

$(5 \cos x - 3)(\cos x + 1) = 0$, so $\cos x = \frac{3}{5}$ or -1

$\cos x = \frac{3}{5}$: $x = 53.1°, 306.9°$

$\cos x = -1$: $x = 180.0°$

Answer: x = 53.1°, 180.0°, 306.9°

10 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 24 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos \theta = \frac{8}{17}$. Find P and Q.

Resolve horizontally: $P \cos \theta = 24$, so $P \times \frac{8}{17} = 24$

$P = 51$

Resolve vertically: $Q = P \sin \theta = 51 \times \frac{15}{17}$

Answer: P = 51 N; Q = 45 N