Day 316 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "4n + 7 is prime for every positive integer n."
Show that this statement is false by finding a counter-example.

Try n = 1, 2, 3, ... until the value is not prime.
n = 2 gives 4 × 2 + 7 = 15
15 = 3 × 5, which is not prime.
Answer: n = 2 (value 15 is not prime)

2 Resultant of perpendicular forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two forces of magnitude 8 N and 15 N act at right angles on a particle.
Find the magnitude of the resultant, and the angle it makes with the 8 N force, to 1 d.p.

Magnitude = $\sqrt{(8^2 + 15^2)} = \sqrt{289}$
$\tan \alpha = 15 \div 8$
Answer: 17 N at 61.9°

3 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the test scores for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Class A: 55 − 40 = 15. Class B: 50 − 25 = 25
(b) Medians: 45 (Class A) and 35 (Class B), so Class A has the higher median
IQRs: Class B has the larger IQR, so its scores are more spread out
Answer: (a) 15 and 25 (b) Class A has the higher median; Class B is more spread out (larger IQR), Class A more consistent

4 Power, driving force and resistance (CIE)

A car of mass 1500 kg moves along a horizontal road. The engine works at a constant rate of 30 kW. The resistance to motion is constant at 600 N.
Find the acceleration when the speed is 8 m s^{-1}.

P = Fv: F = 30000 ÷ 8 = 3750 N
F − R = ma: 3750 − 600 = 1500 a
Answer: 2.1 m s^{-2}

5 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The depth of water at a pier is modelled by $D = 11 + 4 \sin(\frac{\pi t}{6})$, where t is in hours after noon.
Find (a) the greatest and least values, (b) the values of t with $0 \le t \le 12$ when it equals 9 m.

(a) sin ranges from −1 to 1: greatest 11 + 4 = 15, least 11 − 4 = 7
(b) $11 + 4 \sin(\frac{\pi t}{6}) = 9$, so $\sin(\frac{\pi t}{6}) = -\frac{1}{2}$
$\frac{\pi t}{6} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, so t = 7 or 11
Answer: (a) 15 m, 7 m (b) t = 7 and 11

6 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $5^{2x} - 20 \times 5^x - 125 = 0$.

Let $y = 5^x$. Then $5^{2x} = y^2$, so $y^2 - 20y - 125 = 0$
$(y - 25)(y + 5) = 0$
$y = 25$ or $y = -5$
$5^x > 0$ for all x, so $y = -5$ is rejected. $5^x = 25$ gives x = 2.
Answer: x = 2

7 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = t^2$, $y = t^3$.
Find the gradient at $t = -2$ and the equation of the normal there.

$\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 3t^2$

Gradient $= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -3$ at $t = -2$

Point: $(4, -8)$; normal gradient $= \frac{1}{3}$

$y + 8 = \frac{1}{3}(x - 4)$, so $y = \frac{1}{3}x - \frac{28}{3}$

Answer: gradient -3; normal $y = \frac{1}{3}x - \frac{28}{3}$

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $12x + 5y = k$ is a tangent to the circle
$(x - 1)^2 + (y + 2)^2 = 169$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (12, 5).
A unit step in this direction is (12, 5)/13, so the points of contact are
$(1, -2) \pm 13 \times (12, 5)/13 = (1, -2) \pm (12, 5)$
T = (13, 3) or $(-11, -7)$, and $k = 12x + 5y$ gives:
$k = 171$ or $k = -167$
Answer: $k = 171$ or $k = -167$

9 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
$P(X < 88) = 0.1587$ and $P(X > 118) = 0.0668$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.500$
$88 = \mu + (-0.9998)\sigma$ and $118 = \mu + 1.500\sigma$
Subtract: $30 = 2.500\,\sigma$, so $\sigma = 12.0$
$\mu = 88 - (-0.9998)(12.00) = 100$
Answer: $\mu = 100$, $\sigma = 12.0$

10 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $2\cos\theta + 3\sin\theta \equiv \sqrt{13}\cos(\theta - 56.3°)$,
solve $2\cos\theta + 3\sin\theta = -3$ for $0 \le \theta \le 360°$. (1 d.p.)

$\sqrt{13}\cos(\theta - 56.3°) = -3$, so $\cos(\theta - 56.3°) = \frac{-3}{\sqrt{13}}$

$\theta - 56.3° = \pm 146.3°$
$\theta = 56.3° + 146.3° = 202.6°$
$\theta = 56.3° - 146.3° = -90.0°$, so $\theta = 270.0°$
Answer: $\theta = 202.6°, 270.0°$