Day 315 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 14 cm and angle 72°.
Find, in terms of π, the arc length and the area of the sector.

$\theta = 72° = \frac{2\pi}{5}$ radians

Arc length $= r\theta = \frac{28\pi}{5}$ cm

Area $= \frac{1}{2}r^2\theta = \frac{196\pi}{5}$ cm^2

Answer: arc $\frac{28\pi}{5}$ cm, area $\frac{196\pi}{5}$ cm^2

2 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3t^2$, $y = 3t$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve.

From $y = 3t$: $t = \frac{y}{3}$

$x = 3(\frac{y}{3})^2 = \frac{3y^2}{9}$

So $y^2 = 3x$

Answer: $y^2 = 3x$

3 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected vertically upwards with speed 17 m s^{-1} from ground level.
Take $g = 9.8$ m s^{-2}. Find, to 3 significant figures,
(a) the greatest height reached, (b) the time of flight.

(a) $0 = 17^2 - 2 \times 9.8 \times H$, so $H = 289 \div 19.6$
(b) $s = 0$: $t = 2 \times 17 \div 9.8$
Answer: (a) 14.7 m (b) 3.47 s

4 Laws of logarithms: solving equations (All boards)

Solve $\log_3(x + 4) - \log_3(x - 2) = 1$.

$\log_3(\frac{x+4}{x-2}) = 1$, so $\frac{x+4}{x-2} = 3^1 = 3$
$x + 4 = 3(x - 2) = 3x - 6$
$2x = 10$, so $x = 5$
Answer: $x = 5$

5 Independent and mutually exclusive events (All boards)

A and B are independent events with $P(A) = 0.2$ and $P(A \cup B) = 0.8$.
(a) Find P(B).
(b) Find $P(A' \cap B')$.

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$
$0.8 = 0.2 + P(B)(1 - 0.2)$, so $P(B) = 0.6 \div 0.8 = 0.75$
(b) $P(A' \cap B') = 1 - P(A \cup B) = 0.2$
Answer: (a) 0.75 (b) 0.2

Day 315 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point $(-2, 5)$ and B is the point $(0, -4)$.
Find the perpendicular bisector of AB as $ax + by + c = 0$, with integer a, b, c.

Midpoint of AB = $(-1, \frac{1}{2})$

Gradient of AB = $\frac{-9}{2} = -\frac{9}{2}$

Perpendicular gradient = $\frac{2}{9}$

$y - \frac{1}{2} = \frac{2}{9}(x + 1)$, which gives $4x - 18y + 13 = 0$

Answer: $4x - 18y + 13 = 0$

7 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = e^x + 1$ to find the exact value of

$\int_0^{\ln 2} (e^x + 1)^2 e^x \, dx$.

$\frac{du}{dx} = e^x$, so $e^x \, dx = du$

Limits: $x = 0$ gives $u = 2$, $x = \ln 2$ gives $u = 3$

$\int_2^3 u^2 \, du = [\frac{u^3}{3}]_2^3$

$= \frac{19}{3}$

Answer: $\frac{19}{3}$

8 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (3t^2 + 3)\mathbf{i} + (4t - 16)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0
$4t - 16 = 0$, so $t = 4$
$\mathbf{v} = (3 \times 4^2 + 3)\mathbf{i} = 51\mathbf{i}$
Answer: t = 4 s, speed 51 m s^{-1}

9 Normal approximation to the binomial (AQA, OCR, CIE)

$X \sim B(100, 0.25)$. Use a normal approximation to find $P(X \geq 30)$, to 3 significant figures.

$Y \sim N(np, np(1 - p)) = N(25.00, 18.75)$, so $\sigma = \sqrt{18.75} = 4.330$
Use a continuity correction.
$P(X \geq 30) \approx P(Y > 29.5) = P(Z > 1.039) = 0.149$
Answer: 0.149

10 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x + 4y = k$ is a tangent to the circle
$(x + 3)^2 + y^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (3, 4).
A unit step in this direction is (3, 4)/5, so the points of contact are
$(-3, 0) \pm 10 \times (3, 4)/5 = (-3, 0) \pm (6, 8)$
T = (3, 8) or (−9, −8), and $k = 3x + 4y$ gives:
k = 41 or k = −59
Answer: k = 41 or k = −59