Day 314 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-3x + 9 \leq 2x - 9$.

$-5x \leq -18$

Divide by -5 (reversing the inequality): $x \geq \frac{18}{5}$

Answer: $x \geqslant \frac{18}{5}$

2 Product rule (All boards)

Given that $y = x^2e^{-3x}$,

find $\frac{dy}{dx}$, and its value when $x = 0$.

Product rule with $u = x^2$, $v = e^{-3x}$

$u' = 2x$, $v' = -3e^{-3x}$

$\frac{dy}{dx} = u'v + uv' = -x(3x - 2)e^{-3x}$

When $x = 0$: $\frac{dy}{dx} = 0$

Answer: $\frac{dy}{dx} = -x(3x - 2)e^{-3x}$; value 0

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 3x - 6$ and $g(x) = x^2 + 6$ for $x \in \mathbb{R}$.

(a) Find $fg(x)$.

(b) Solve $gf(x) = 15$.

$fg(x) = f(x^2 + 6) = 3(x^2 + 6) - 6 = 3x^2 + 12$

$gf(x) = (3x - 6)^2 + 6 = 15$

$(3x - 6)^2 = 9$, so $3x - 6 = \pm 3$

$x = 1$ or $x = 3$

Answer: (a) $fg(x) = 3x^2 + 12$ (b) $x = 1$ or $x = 3$

4 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 20 kW. The resistance to motion is constant at 500 N.

Find the acceleration when the speed is 10 m s^{-1}.

$P = Fv$: $F = 20000 \div 10 = 2000$ N

$F - R = ma$: $2000 - 500 = 1000a$

Answer: 1.5 m s^{-2}

5 Inverse normal (All boards)

A battery lasts X hours, normal with mean 45, s.d. 0.5.

Find k (3 s.f.): (a) $P(X < k) = 0.25$,

(b) $P(X > k) = 0.01$.

(a) $z = -0.6745$, so $k = 45 + (-0.6745) \times 0.5 = 44.7$

(b) $P(X < k) = 0.99$, $z = 2.326$, so $k = 45 + 2.326 \times 0.5 = 46.2$

Answer: (a) 44.7 (b) 46.2

6 Laws of logarithms: solving equations (All boards)

Solve $2 \ln x = \ln(x + 6)$.

$\ln x^2 = \ln(x + 6)$, so $x^2 = x + 6$

$x^2 - x - 6 = 0$, so $(x - 3)(x + 2) = 0$

$x > 0$ for $\ln x$ to exist, so $x = 3$

Answer: $x = 3$

Day 314 **A Level Maths: B Tier 10-a-day: Model Solutions**

7 Binomial critical region (All boards)

$X \sim B(20, p)$. H_0: $p = 0.25$ is to be tested against H_1: $p < 0.25$ at the 10% significance level.
Find the critical region and the actual significance level.

Under H_0, $X \sim B(20, 0.25)$
$P(X \leq 2) = 0.0913 \leq 0.1$
$P(X \leq 3) = 0.2252 > 0.1$
The critical region is $X \leq 2$
Answer: $X \leq 2$; actual significance level 9.13%

8 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{3x^2 + 11x - 38}{(x-1)(x+5)}$ in the form $P + \frac{Q}{(x-1)} + \frac{R}{(x+5)}$.

$3x^2 + 11x - 38 = 3(x-1)(x+5) + (-x - 23)$, so $P = 3$
$\frac{3x^2 + 11x - 38}{(x-1)(x+5)} \equiv 3 + \frac{Q}{(x-1)} + \frac{R}{(x+5)}$
$x = 1$ gives $Q = -4$; $x = -5$ gives $R = 3$
Answer: $3 + -\frac{4}{(x-1)} + \frac{3}{(x+5)}$

9 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of
$\int_0^{\frac{\pi}{4}} (2x - 1)\cos 2x \, dx$.

Let $u = 2x - 1$ and $\frac{dv}{dx} = \cos 2x$, so $\frac{du}{dx} = 2$ and $v = \frac{\sin 2x}{2}$
Integral $= [uv] - \int v\frac{du}{dx} \, dx$
$= [\frac{(2x-1)\sin 2x}{2}]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin 2x \, dx$
$= \frac{\pi}{4} - 1$
Answer: $\frac{\pi}{4} - 1$

10 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 24t - 4t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 24 - 8t$
Maximum speed when $a = 0$: $t = 24 \div 8 = 3$
$v = 24 \times 3 - 4 \times 3^2$
Answer: 36 m s^{-1} at t = 3 s