**Day 313** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line $l_1$ has equation $7x - 3y = -21$ and line $l_2$ has equation $7x + 2y = 14$.
The lines meet at P. Find the area of the triangle formed by $l_1$, $l_2$ and the x-axis.

Solve simultaneously: P = (0, 7)
$l_1$ meets y = 0 at $x = -3$, and $l_2$ meets y = 0 at x = 2
Base = 5, height = 7
Area = $\frac{1}{2} \times$ base $\times$ height = $\frac{35}{2}$

**Answer:** $\frac{35}{2}$

## 2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 96; systematic sample of size 8.
(a) Find the sampling interval.
(b) The first item is number 1. Find the 6-th item.

(a) Interval = 96 ÷ 8 = 12
(b) 6-th item = 1 + 5 × 12 = 61
**Answer: interval 12; item number 61**

## 3 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 10 s, (b) the distance between the stations.

(a) a = gradient = 8 ÷ 10
(b) Distance = area of trapezium = ½ × (25 + 10) × 8
**Answer: (a) 0.8 m s$^{-2}$ (b) 140 m**

## 4 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = \frac{4y}{x}$ for $x > 0$,
given that y = 4 when x = 1.

$\frac{1}{y}\,dy = \frac{4}{x}\,dx$, so $\ln y = 4 \ln x + c$
x = 1, y = 4: $c = \ln 4$
$y = 4x^4$
**Answer: $y = 4x^4$**

## 5 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x + 2)(x - 3)(x + 5) \geq 0$.

Critical values: $x = -5$, $x = -2$, $x = 3$
The cubic has a positive $x^3$ term, so the graph goes from bottom left to top right
and crosses the axis at each critical value.
Reading from the sketch: $-5 \leq x \leq -2$ or $x \geq 3$
**Answer: $-5 \leq x \leq -2$ or $x \geq 3$**

## 6 Binomial: smallest n (All boards)

Given that each component fails independently with probability 0.2, find the least number of components needed for the probability of at least one success to exceed 0.95.

P(at least one) = $1 - 0.8^n > 0.95$
$0.8^n < 0.05$
$n > \frac{\log 0.05}{\log 0.8} = 13.43$
**Answer: n = 14**

**Day 313** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 7 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (3t^2 + 5)\mathbf{i} + (2t - 6)\mathbf{j}$ m s$^{-1}$.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0
$2t - 6 = 0$, so $t = 3$
$\mathbf{v} = (3 \times 3^2 + 5)\mathbf{i} = 32\mathbf{i}$
**Answer: t = 3 s, speed 32 m s$^{-1}$**

## 8 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 2t + 1$, $y = t^2 - t - 1$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve in the form $y = f(x)$.

From the first equation: $t = \frac{x - 1}{2}$

Substitute t into $y = t^2 - t - 1$ and expand.
$y = \frac{1}{4}x^2 - x - \frac{1}{4}$
**Answer: $y = \frac{1}{4}x^2 - x - \frac{1}{4}$**

## 9 Intersection of two lines in 3D (CIE)

Line $l_1$: $\mathbf{r} = \begin{pmatrix}1\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\0\end{pmatrix}$.
Line $l_2$: $\mathbf{r} = \begin{pmatrix}3\\0\\1\end{pmatrix} + \mu\begin{pmatrix}0\\-2\\-1\end{pmatrix}$.
Show that $l_1$ and $l_2$ intersect and find the point.

i: $1 + 2\lambda = 3 + 0\mu$
j: $1 + 1\lambda = 0 + (-2)\mu$
Solving i and j: $\lambda = 1$, $\mu = -1$
k check: $2 + 0(1) = 2$ and $1 + (-1)(-1) = 2$, equal
Point: $\begin{pmatrix}3\\2\\2\end{pmatrix}$
**Answer:** $\begin{pmatrix}\mathbf{3}\\\mathbf{2}\\\mathbf{2}\end{pmatrix}$

## 10 Area under a curve (All boards)

The diagram shows the curve $y = x(6 - x)$ and the line $y = 2x$.
Find the exact area of the shaded region R.

Intersect: $x(6 - x) = 2x$, so $x = 0$ or $x = 4$
Area $= \int_0^4 (x(6 - x) - 2x)\,dx = \int_0^4 (4x - x^2)\,dx$
$= [2x^2 - \frac{x^3}{3}]_0^4 = \frac{32}{3}$
**Answer: $\frac{32}{3}$**