Day 311 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quotient rule (All boards)

Find the x-coordinate(s) of the stationary point(s) of the curve $y = \frac{e^x}{x}$.

$\frac{dy}{dx} = \frac{e^x}{x} - \frac{e^x}{x^2}$

Stationary points where $\frac{dy}{dx} = 0$, i.e. the numerator is zero

Numerator: $(x - 1)e^x = 0$

$x = 1$

Answer: x = 1

2 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is (5, −4) and B is (−3, −12).
Find an equation of the circle in the form
$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = (1, −8)

Radius$^2 = (5 - 1)^2 + (-4 - (-8))^2 = 32$

Answer: $(x - 1)^2 + (y + 8)^2 = 32$

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 5 cm and area $\frac{25\pi}{4}$ cm^2.

Find the angle of the sector in degrees.

$\frac{1}{2}r^2\theta = \frac{25\pi}{4}$, so $\theta = \frac{2 \times \frac{25\pi}{4}}{5^2} = \frac{\pi}{2}$ radians

$\theta = \frac{\pi}{2} \times \frac{180}{\pi} = 90°$

Answer: 90°

4 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 13 pairs of values, $\bar{x} = 8$, $\bar{y} = 20$, $S_{xx} = 50$ and $S_{xy} = 125$.
(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Estimate y when x = 12.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{125}{50} = 2.5$

$a = \bar{y} - b\bar{x} = 20 - 2.5 \times 8 = 0$

$y = 0 + 2.5x$

(b) $y = 0 + 2.5 \times 12 = 30$

Answer: y = 0 + 2.5x; y = 30

5 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a} \cdot \mathbf{b} = (5)(-1) + (-2)(2) + (4)(-2) = -17$

$|\mathbf{a}| = \sqrt{45}$, $|\mathbf{b}| = \sqrt{9}$

$\cos\theta = \frac{-17}{\sqrt{45}\sqrt{9}} = -0.8447$

$\theta = 147.6°$

Answer: 147.6°

6 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x + 1| = |4x + 1|$.

Either $2x + 1 = 4x + 1$ or $2x + 1 = -(4x + 1)$

$2x + 1 = 4x + 1$ gives $x = 0$

$2x + 1 = -(4x + 1)$ gives $x = -\frac{1}{3}$

Answer: $x = -\frac{1}{3}$ and $x = 0$

7 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass, X g, of a loaf is normally distributed with mean 60 and s.d. 5.
A random sample of 25 is taken. Find the probability that the mean mass is more than 61.5 (3 s.f.).

$\overline{X} \sim N(60, \frac{5^2}{25})$, standard deviation 1.000

$z = \frac{61.5 - 60}{1.000} = 1.500$

$P = 0.0668$

Answer: 0.0668

8 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (3t^2 + 4)\mathbf{i} + (4t - 16)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0

$4t - 16 = 0$, so $t = 4$

$\mathbf{v} = (3 \times 4^2 + 4)\mathbf{i} = 52\mathbf{i}$

Answer: t = 4 s, speed 52 m s^{-1}

9 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = -x$ for $1 < x \leq 5$.
Find the range of f.

f is decreasing, so the extreme values occur at the ends of the domain.

$f(1) = -1$ and $f(5) = -5$

$x = 1$ is excluded and $x = 5$ is included, so $f = -1$ is excluded and $f = -5$ is included.

Answer: $-5 \leq f(x) < -1$

10 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown horizontally at 28 m s^{-1} from the top of a building 44.1 m above level ground.
Take g = 9.8 m s^{-2}.
Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $44.1 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 9$

$t = 3$ s

Horizontal: $x = 28 \times 3$

Answer: 3 s, 84 m