**Day 310** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = -t + 3$, $y = 3t^2 + 2t + 4$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve in the form $y = f(x)$.

From the first equation: $t = \frac{x - 3}{-1}$

Substitute t into $y = 3t^2 + 2t + 4$ and expand.
$y = 3x^2 - 20x + 37$
**Answer: $y = 3x^2 - 20x + 37$**

## 2 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = t^3 - 4t^2$ metres.
Find its velocity and acceleration when $t = 3$.

$v = ds/dt = 3t^2 - 8t$
$a = dv/dt = 6t - 8$
$t = 3$: $v = 3$, $a = 10$
**Answer: $v = 3$ m s$^{-1}$, $a = 10$ m s$^{-2}$**

## 3 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} -4 \\ 1 \\ 5 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 1 \\ -4 \\ 10 \end{pmatrix}$.
The point P lies on AB with AP : PB = 3 : 2.
Find the position vector of P.

$\text{AB} = \begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$

$\text{AP} = \frac{3}{5}\text{AB} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} -1 \\ -2 \\ 8 \end{pmatrix}$

**Answer:** $\begin{pmatrix} \mathbf{-1} \\ \mathbf{-2} \\ \mathbf{8} \end{pmatrix}$

## 4 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the times taken (minutes) to complete a puzzle for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Class A: $50 - 20 = 30$. Class B: $50 - 35 = 15$
(b) Medians: 35 (Class A) and 40 (Class B), so Class B has the higher median
IQRs: Class A has the larger IQR, so its scores are more spread out
**Answer: (a) 30 and 15 (b) Class B has the higher median; Class A is more spread out (larger IQR), Class B more consistent**

## 5 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{-2x+3} = 4^{-3x+1}$.

Write both sides as powers of 2: $2^{-2x+3} = 2^{2(-3x+1)}$
Equate the indices: $-2x + 3 = -6x + 2$
$4x = -1$
$x = -\frac{1}{4}$

**Answer: $x = -\frac{1}{4}$**

**Day 310** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 6 Union, conditional probability and independence (All boards)

P(A) = 0.4, P(B) = 0.3, P(A ∩ B) = 0.07.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) $P(A \cup B) = 0.4 + 0.3 - 0.07 = 0.63$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.07}{0.3} = \frac{7}{30}$

(c) $P(A)P(B) = 0.4 \times 0.3 = 0.12$, which does not equal $P(A \cap B)$

**Answer: (a) 0.63 (b) $\frac{7}{30}$ (c) not independent**

## 7 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a} \cdot \mathbf{b} = (-2)(-1) + (0)(3) + (3)(5) = 17$

$|\mathbf{a}| = \sqrt{13}$, $|\mathbf{b}| = \sqrt{35}$

$\cos\theta = \frac{17}{\sqrt{13}\sqrt{35}} = 0.7970$

$\theta = 37.2°$

**Answer: 37.2°**

## 8 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $4x^2 + kx + 4 = 0$ has equal roots.
Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$

$k^2 - 4 \times 4 \times 4 = 0$

$k^2 = 64$

$k = \pm 8$

**Answer: k = ±8**

## 9 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trunk of mass 15 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{12}{13}$.

The coefficient of friction is 0.2. Take g = 9.8 m s$^{-2}$. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 147 - \frac{5}{13}T$

Horizontal: $T\cos\alpha = \mu R$: $\frac{12}{13}T = 0.2(147 - \frac{5}{13}T)$

$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.2 \times 15 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)

**Answer: 29.4 N**

## 10 Equations using $\sin^2x + \cos^2x = 1$ (All boards)

Solve $4\sin^2x - \cos x - 1 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2x = 1 - \cos^2x$: $3 - 4\cos^2x - \cos x = 0$, i.e. $4\cos^2x + \cos x - 3 = 0$

$(\cos x + 1)(4\cos x - 3) = 0$, so $\cos x = -1$ or $\frac{3}{4}$

$\cos x = -1$: $x = 180.0°$

$\cos x = \frac{3}{4}$: $x = 41.4°, 318.6°$

**Answer: x = 41.4°, 180.0°, 318.6°**