Day 31 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 2)^2 + (y + 4)^2 = 13$.
The point P(0, −7) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (2, −4). Gradient of radius CP = $\frac{-3}{-2} = \frac{3}{2}$

The tangent is perpendicular to the radius: gradient = $-\frac{2}{3}$

$y + 7 = -\frac{2}{3}x$

Multiply out and collect terms: 2x + 3y + 21 = 0
Answer: 2x + 3y + 21 = 0

2 Trig equations in radians (All boards)

Solve tan x = −4 for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

tan θ = −4 for $0 \leq x \leq 2\pi$
Calculator: x = −1.326, then use the symmetry of the graph
x = 1.816, 4.957
Answer: x = 1.82, 4.96

3 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

30 pairs of rainfall and crop yield give r = −0.217.
One-tailed critical value (5%): 0.3061.
Test for negative correlation.

H_0: ρ = 0, H_1: ρ < 0
r = −0.217, critical value −0.3061; r is not beyond the critical value in the right direction
r is not in the critical region, so do not reject H_0
Conclusion: there is insufficient evidence of negative correlation between rainfall and crop yield
Answer: do not reject H_0: there is insufficient evidence of negative correlation between rainfall and crop yield

4 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sum_{r=1}^{n}(4r - 3) = 231$, find the value of n.

Sum = $\frac{n}{2}(1 + 4n - 3) = 231$

$4n^2 - 2n - 462 = 0$
$2(n - 11)(2n + 21) = 0$
n must be positive, so n = 11
Answer: n = 11

5 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

P(X = x) = k(x + 2) for x = 1, 2, ..., 3.
(a) Find k.
(b) Find E(X) and Var(X).

(a) k(3 + 4 + 5) = 1, so 12k = 1 and k = $\frac{1}{12}$

(b) E(X) = $1 \times \frac{1}{4} + 2 \times \frac{1}{3} + 3 \times \frac{5}{12} = \frac{13}{6}$

$E(X^2) = \frac{16}{3}$, so Var(X) = $\frac{16}{3} - (\frac{13}{6})^2 = \frac{23}{36}$

Answer: (a) k = $\frac{1}{12}$ (b) E(X) = $\frac{13}{6}$, Var(X) = $\frac{23}{36}$

6 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 70 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 20 to t = 50.
The floors are 400 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (70 + 30) × V = 400
100 V = 800 so V = 8
Deceleration = 8 ÷ 20
Answer: V = 8; deceleration 0.4 m s^{-2}

7 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $2\cos^2 x - \sin x - 2 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $-2\sin^2 x - \sin x = 0$, i.e. $2\sin^2 x + \sin x = 0$
$(\sin x)(2\sin x + 1) = 0$, so $\sin x = 0$ or $-\frac{1}{2}$
$\sin x = 0$: x = 0.0°, 180.0°, 360.0°
$\sin x = -\frac{1}{2}$: x = 210.0°, 330.0°
Answer: x = 0.0°, 180.0°, 210.0°, 330.0°, 360.0°

8 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A javelin is projected from level ground with horizontal velocity component 23 m s^{-1} and vertical velocity component 44.1 m s^{-1} upwards.
Take g = 9.8 m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 44.1\,t - 4.9t^2$, so t = 44.1 ÷ 4.9
(b) Range = horizontal speed × time = 23 × 9
Answer: (a) 9 s (b) 207 m

9 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 - x)^{-2}$ up to the x^2 term.

(b) Use x = 0.04 to estimate $\frac{1}{0.96^2}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with n = −2, a = −1:
$= 1 + 2x + 3x^2$
(b) 1 − 1 × 0.04 = 0.96, so substitute x = 0.04:
$1 + 2 \times 0.04 + 3 \times 0.04^2 = 1.08480$
Answer: (a) $1 + 2x + 3x^2 + \ldots$ (b) 1.08480

10 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$ and B has position vector $\begin{pmatrix} -4 \\ -5 \end{pmatrix}$.
The point P lies on AB with AP : PB = 2 : 1.
Find the position vector of P.

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$
$\overrightarrow{AP} = \frac{2}{3}\overrightarrow{AB} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$
$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$
Answer: $\begin{pmatrix} \mathbf{-4} \\ \mathbf{-3} \end{pmatrix}$