Day 309 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A puck of mass 2 kg is moving on a smooth horizontal surface at 1 m s^{-1} when a constant horizontal force of 20 N, in the direction of motion, acts for 2 s.
Find the speed of the puck at the end of this time and the distance it moves.

F = ma: a = 20 ÷ 2 = 10
v = u + at = 1 + 10 × 2
$s = ut + \frac{1}{2}at^2$
Answer: 21 m s^{-1}; 22 m

2 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - x > 30$.

Rearrange: $x^2 - x - 30 > 0$
Factorise: $(x - 6)(x + 5) > 0$
Critical values: $x = -5$ and $x = 6$
Sketch the ∪-shaped parabola: $x < -5$ or $x > 6$
Answer: $x < -5$ or $x > 6$

3 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{21}{29}$ and $\cos B = \frac{9}{41}$, where A and B are acute,
find the exact value of sin(A + B).

$\cos A = \frac{20}{29}$ and $\sin B = \frac{40}{41}$
sin(A + B) = sin A cos B + cos A sin B
$= \frac{21}{29} \times \frac{9}{41} + \frac{20}{29} \times \frac{40}{41} = \frac{989}{1189}$
Answer: $\frac{989}{1189}$

4 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations
$x = -1 + 5\cos t$, $y = 2 + 5\sin t$, for $0 \leq t < 2\pi$. Find a Cartesian equation of the curve.

$\cos t = \frac{x + 1}{5}$ and $\sin t = \frac{y - 2}{5}$
Use $\cos^2 t + \sin^2 t = 1$:
$(x + 1)^2 + (y - 2)^2 = 25$
Answer: $(x + 1)^2 + (y - 2)^2 = 25$

5 Binomial one-tailed test (All boards)

A manufacturer claims that a proportion 0.25 of its sweets are red. A sample of 20 sweets has 2 red.
Test, at the 5% level, whether the true proportion is lower than claimed.

H_0: p = 0.25, H_1: p < 0.25. Let X ~ B(20, 0.25) under H_0
$P(X \leq 2) = 0.0913$
0.0913 > 0.05, so do not reject H_0
There is insufficient evidence that the proportion is lower than claimed
Answer: do not reject H_0: insufficient evidence that the proportion is lower

6 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = ab^x$. A graph of ln y against x is a straight line with gradient ln 10 that passes through (0, ln 4).
Find the values of a and b.

ln y = ln a + x ln b
gradient = ln b = ln 10, so b = 10
intercept ln a = ln 4, so a = 4
Answer: a = 4, b = 10

7 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 62) = 0.0668 and P(X > 92) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$
$62 = \mu + (-1.500)\sigma$ and $92 = \mu + 0.9998\sigma$
Subtract: $30 = 2.500\ \sigma$, so $\sigma = 12.0$
$\mu = 62 - (-1.500)(12.00) = 80.0$
Answer: μ = 80.0, σ = 12.0

8 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 20°C: $\frac{dT}{dt} = -k(T - 20)$. Initially T = 80 and after 6 minutes T = 55.
Find k to 3 s.f. and the time to cool to 30°C, to 3 s.f.

$\frac{1}{T - 20}\,dT = -k\,dt$, so $\ln(T - 20) = -kt + c$, i.e. $T = 20 + 60e^{-kt}$

$t = 6$: $55 = 20 + 60e^{-6k}$, so $e^{-6k} = \frac{7}{12}$

$k = \frac{1}{6}\ln\frac{60}{35} = 0.08983$

$30 = 20 + 60e^{-kt}$: $t = \frac{1}{k}\ln\frac{60}{10} = 19.95$

Answer: k = 0.0898, t = 19.9 minutes

9 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards at 14.7 m s^{-1} from a point 19.6 m above horizontal ground.
Take g = 9.8 m s^{-2}.
Find the time before it hits the ground.

Taking up as positive: $-19.6 = 14.7\,t - 4.9t^2$
$4.9t^2 - 14.7\,t - 19.6 = 0$
$t = (14.7 \pm \sqrt{600.25}) \div 9.8 = (14.7 \pm 24.5) \div 9.8$
Positive root only.
Answer: 4 s

10 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $81^{\frac{3}{4}}$.

$81 = 3^4$, so $81^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^3$
$= 27$
Answer: 27