Day 308 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 240; systematic sample of size 24.
(a) Find the sampling interval.
(b) The first item is number 5. Find the 5-th item.

(a) Interval = 240 ÷ 24 = 10
(b) 5-th item = 5 + 4 × 10 = 45
Answer: interval 10; item number 45

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $5x - 8 \geq -3x - 7$.

$8x \geq 1$

Divide by 8: $x \geq \frac{1}{8}$

Answer: $x \geq \frac{1}{8}$

3 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -8 \\ -8 \\ 4 \end{pmatrix}$.
Find $|\mathbf{a}|$ and a unit vector in the direction of a.

$|\mathbf{a}| = \sqrt{8^2 + 8^2 + 4^2} = \sqrt{144} = 12$
Unit vector $= \frac{1}{12}\mathbf{a}$

Answer: $|\mathbf{a}| = 12$, unit vector $\frac{1}{12}\begin{pmatrix} -8 \\ -8 \\ 4 \end{pmatrix}$

4 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 50 and common ratio 0.9.
Find the smallest n for which the nth term is less than 1.

$50 \times 0.9^{n-1} < 1$
$(n - 1) \lg 0.9 < \lg(1 \div 50)$
lg 0.9 is negative, so dividing reverses the inequality:
$n - 1 > \lg(1 \div 50) \div \lg 0.9 = 37.130$, so $n > 38.130$
The smallest integer n is 39
Answer: n = 39

5 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 36 kW. The resistance to motion is constant at 500 N.
Find the acceleration when the speed is 8 m s^{-1}.

P = Fv: F = 36000 ÷ 8 = 4500 N
F − R = ma: 4500 − 500 = 1000 a
Answer: 4 m s^{-2}

6 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre (5, 3) and passes through the point (2, −1).
Find an equation of the circle.

$r^2 = (2 - 5)^2 + (-1 - 3)^2$
$= 9 + 16 = 25$
Answer: $(x - 5)^2 + (y - 3)^2 = 25$

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7 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 12t - 2t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 12 - 4t$

Maximum speed when $a = 0$: $t = 12 \div 4 = 3$

$v = 12 \times 3 - 2 \times 3^2$

Answer: 18 m s^{-1} at t = 3 s

8 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 8 \\ -10 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $4 + (-2)\lambda = 8 + (-2)\mu$

j: $-3 + 2\lambda = -10 + 3\mu$

Solving i and j: $\lambda = 1$, $\mu = 3$

k check: $2 + 0(1) = 2$ and $-4 + 2(3) = 2$, equal

Point: $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{2} \\ \mathbf{-1} \\ \mathbf{2} \end{pmatrix}$

9 Binomial two-tailed test (All boards)

A factory says a proportion 0.3 of its parts are defective. In a sample of 50 parts, 19 are defective.
Test, at the 5% significance level, whether the proportion is different from 0.3.

H_0: $p = 0.3$, H_1: $p \neq 0.3$. Under H_0, $X \sim B(50, 0.3)$

$P(X \geq 19) = 1 - P(X \leq 18) = 0.1406$

Two-tailed: compare with 0.025. $0.1406 > 0.025$, so do not reject H_0

There is insufficient evidence that the proportion is different from 0.3

Answer: do not reject H_0: insufficient evidence that the proportion is different from 0.3

10 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB subtends an angle of 1.2 radians at the centre O of a circle of radius 8 cm.
Find the perimeter of the minor segment, to 3 s.f.

Arc $= r\theta = 8 \times 1.2 = 9.6$

Chord AB $= 2r \sin(\frac{\theta}{2}) = 9.034$

Perimeter $= 9.6 + 9.034 = 18.63$ cm

Answer: 18.6 cm