Day 307 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = \frac{1}{(x^2 + 3)}$,

find $\frac{dy}{dx}$, and the gradient of the curve when x = 0.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = -\frac{2x}{(x^2 + 3)^2}$

When x = 0: gradient = 0

Answer: $\frac{dy}{dx} = -\frac{2x}{(x^2 + 3)^2}$; gradient 0

2 Binomial probabilities (All boards)

X ~ B(10, 0.7). Find, to 3 significant figures,

(a) $P(4 \leq X \leq 7)$

(b) $P(X \geq 7)$

(a) $P(4 \leq X \leq 7) = P(X \leq 7) - P(X \leq 3)$

(a) = 0.607

(b) $P(X \geq 7) = 1 - P(X \leq 6)$

(b) = 0.650

Answer: (a) 0.607 (b) 0.650

3 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sector OAB of a circle with centre O, radius 8 cm and angle 1.3 radians.

Find (a) the arc length AB, (b) the area of the sector.

Arc length = $r\theta = 8 \times 1.3 = 10.4$ cm

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times 1.3 = 41.6$ cm^2

Answer: (a) 10.4 cm (b) 41.6 cm^2

4 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x + 1$ and $g(x) = x^2 - 6$ for $x \in \mathbb{R}$.

(a) Find fg(x).

(b) Solve gf(x) = −5.

$fg(x) = f(x^2 - 6) = 2(x^2 - 6) + 1 = 2x^2 - 11$

$gf(x) = (2x + 1)^2 - 6 = -5$

$(2x + 1)^2 = 1$, so $2x + 1 = \pm 1$

$x = -1$ or $x = 0$

Answer: (a) $fg(x) = 2x^2 - 11$ (b) $x = -1$ or $x = 0$

5 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is kicked from horizontal ground with speed 63.7 m s^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{5}{12}$.

Take g = 9.8 m s^{-2}.

Find (a) the time of flight, (b) the horizontal range.

cos θ = 12/13, sin θ = 5/13: horizontal 58.8, vertical 24.5

(a) $0 = 24.5\,t - 4.9t^2$, $t = 24.5 \div 4.9$

(b) Range = 58.8 × 5

Answer: (a) 5 s (b) 294 m

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6 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.5. In a sample of 25 students, 6 pass first time.

Test, at the 5% significance level, whether the probability is different from 0.5.

H_0: p = 0.5, H_1: p ≠ 0.5. Under H_0, X ~ B(25, 0.5)

P(X ≤ 6) = 0.0073

Two-tailed: compare with 0.025. 0.0073 < 0.025, so reject H_0

There is sufficient evidence that the probability is different from 0.5

Answer: reject H_0: sufficient evidence that the probability is different from 0.5

7 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 - x + 5$.

C is stretched by scale factor 2 parallel to the y-axis.

Find its new equation.

Stretch parallel to the y-axis: multiply y by the scale factor.

When stretched by scale factor 2 parallel to the y-axis: $y = 2x^2 - 2x + 10$

Answer: $y = 2x^2 - 2x + 10$

8 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 45 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 20 to t = 40.

The floors are 195 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (45 + 20) × V = 195

65 V = 390 so V = 6

Deceleration = 6 ÷ 5

Answer: V = 6; deceleration 1.2 m s^{-2}

9 Locating roots: change of sign (All boards)

The function f(x) = ln x + x − 5.

Show that f(x) = 0 has a root between x = 3.5 and x = 4.

f(3.5) = −0.247

f(4) = 0.386

f is continuous and changes sign, so there is a root in the interval

Answer: f(3.5) < 0 and f(4) > 0: sign change

10 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -\frac{3}{2}x + 12$. The point P is (7, 8).

(a) Find the line through P perpendicular to l, as ax + by + c = 0 with integer a, b, c.

(b) Find the foot F and the exact distance PF.

(a) Gradient of l = $-\frac{3}{2}$, so the perpendicular has gradient $\frac{2}{3}$

$y - 8 = \frac{2}{3}(x - 7)$, giving $2x - 3y + 10 = 0$

(b) Solve $y = -\frac{3}{2}x + 12$ with $2x - 3y + 10 = 0$: F = (4, 6)

$PF^2 = 3^2 + 2^2 = 13$

Answer: (a) $2x - 3y + 10 = 0$ (b) F(4, 6), PF = $\sqrt{13}$