Day 306 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of 5 consecutive odd integers is always a multiple of 5.

Let the integers be 2n + 1, 2n + 3, 2n + 5, 2n + 7, 2n + 9
Sum = 10n + 25
= 5(2n + 5)
n is an integer, so (2n + 5) is an integer, so the sum is a multiple of 5.
Answer: sum = 5(2n + 5), a multiple of 5

2 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -9 \\ -2 \\ -6 \end{pmatrix}$.

Find |a| and a unit vector in the direction of a.

$|\mathbf{a}| = \sqrt{9^2 + 2^2 + 6^2} = \sqrt{121} = 11$

Unit vector = $\frac{1}{11}\mathbf{a}$

Answer: |a| = 11, unit vector $\frac{1}{11}\begin{pmatrix} -9 \\ -2 \\ -6 \end{pmatrix}$

3 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, goals scored by a player in 11 seasons:
14, 17, 23, 23, 29, 31, 33, 33, 35, 35, 40.
Find Q_1, Q_3, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = 23$, $Q_3 = 35$, IQR = 12
Lower limit = 23 − 1.5 × 12 = 5
Upper limit = 35 + 1.5 × 12 = 53
No value lies outside these limits.
Answer: $Q_1 = 23$, $Q_3 = 35$, IQR = 12; no outliers

4 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 9 and the coefficient of x^2 is 36.
Find the values of n and k.

nk = 9, so $k = \frac{9}{n}$

$\frac{n(n-1)}{2}k^2 = 36$, so $\frac{n(n-1)}{2} \times \frac{81}{n^2} = 36$

$\frac{(n-1) \times 81}{2n} = 36$, so $81(n-1) = 72n$

n = 9, and k = 1
Answer: n = 9, k = 1

5 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf $f(x) = kx^4$ for $0 \leq x \leq 2$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 1). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^5/5]$ from 0 to 2 = $k \times \frac{2^5}{5} = 1$, so $k = \frac{5}{32}$

(b) $P(X > 1) = \int_1^2 \frac{5}{32}x^4\,dx = 1 - \frac{1^5}{2^5} = \frac{31}{32}$

(c) $E(X) = \int_0^2 x\,f(x)\,dx = \frac{5}{32} \times \frac{2^6}{6} = \frac{5}{3}$

Answer: (a) k = $\frac{5}{32}$ **(b)** $\frac{31}{32}$ **(c)** $\frac{5}{3}$

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6 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 10 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{5}{13}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 10$, so $P \times \frac{5}{13} = 10$

$P = 26$

Resolve vertically: $Q = P\sin\theta = 26 \times \frac{12}{13}$

Answer: P = 26 N; Q = 24 N

7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $21\cos\theta + 20\sin\theta \equiv 29\cos(\theta - 43.6°)$,
solve $21\cos\theta + 20\sin\theta = 3.5$ for $0 \leq \theta \leq 360°$. (1 d.p.)

$29\cos(\theta - 43.6°) = 3.5$, so $\cos(\theta - 43.6°) = \frac{3.5}{29}$

$\theta - 43.6° = \pm 83.1°$

$\theta = 43.6° + 83.1° = 126.7°$

$\theta = 43.6° - 83.1° = -39.5°$, so $\theta = 320.5°$

Answer: θ = 126.7°, 320.5°

8 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 3x + 16$ meets the circle $(x - 3)^2 + y^2 = 65$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = 3x + 16$: $(x - 3)^2 + (3x + 16)^2 = 65$

$x^2 + 9x + 20 = 0$

$(x + 4)(x + 5) = 0$, so $x = -5$ or $x = -4$

$y = 3x + 16$: A = (−5, 1), B = (−4, 4)

$AB^2 = 1^2 + 3^2 = 10$

Answer: A(−5, 1), B(−4, 4), AB = $\sqrt{10}$

9 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 4 m and mass 30 kg rests horizontally on supports at C and D, where AC = 1.5 m and DB = 1.5 m.
A girl of mass 10 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero

Moments about D: $30\text{ g} \times (2.5 - 2) = 10\text{ g} \times x$

$30 \times 0.5 = 10x$

Answer: 1.5 m

10 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 5\sec^2 5x\,dx$,

(b) the exact value of $\int_0^{\frac{\pi}{20}} 5\sec^2 5x\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\tan 5x + c$

(b) $[\tan 5x]_0^{\frac{\pi}{20}}$

$= 1$

Answer: (a) tan 5x + c (b) 1