Day 305 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 14.7 m s^{-1} from a point 49 m above horizontal ground.
Take g = 9.8 m s^{-2}.
Find the time before it hits the ground.

Taking up as positive: $-49 = 14.7\,t - 4.9t^2$
$4.9t^2 - 14.7\,t - 49 = 0$
$t = (14.7 \pm \sqrt{1176.49}) \div 9.8 = (14.7 \pm 34.3) \div 9.8$
Positive root only.
Answer: 5 s

2 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 4 \\ -5 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ q \\ -3 \end{pmatrix}$ respectively.
Given that A, B, C are collinear, find q.

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 2 \\ q - -3 \\ -2 \end{pmatrix}$
$\overrightarrow{AC} = \lambda\overrightarrow{AB}$: component 1 gives $2 = \lambda \times 1$, so $\lambda = 2$
$q - -3 = 2 \times -2$, so $q = -7$
Answer: q = −7

3 Work-energy: sliding to rest (CIE)

A block of mass 4 kg is projected along a rough horizontal surface with speed 14 m s^{-1} and comes to rest after sliding 25 m.
Take g = 9.8 m s^{-2}.
Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE
$\mu \times 4 \times 9.8 \times 25 = \frac{1}{2} \times 4 \times 14^2$
$980\,\mu = 392$
Answer: 0.4

4 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 4t$, $y = \frac{7}{t}$, $t \neq 0$.
Find a Cartesian equation of the curve.

Multiply the two equations: $xy = 4t \times \frac{7}{t} = 28$
So $y = \frac{28}{x}$
Answer: $y = \frac{28}{x}$

5 Two independent binomials (All boards)

X ~ B(12, 0.5) and Y ~ B(7, 0.3) are independent.
Find P(X = 5 and Y ≤ 1), to 3 significant figures.

P(X = 5) = 0.1934
P(Y ≤ 1) = 0.3294
Independent, so multiply: 0.1934 × 0.3294 = 0.0637
Answer: 0.0637

6 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:
(a) y = 2f(x) (b) y = f(x) + 5

(a) y = 2f(x): A(−1, 3) maps to (−1, 6), B(3, −5) maps to (3, −10)
(b) y = f(x) + 5: A(−1, 3) maps to (−1, 8), B(3, −5) maps to (3, 0)
Answer: (a) (−1, 6), (3, −10) (b) (−1, 8), (3, 0)

7 Probability without replacement (All boards)

A bag has 8 red and 5 blue counters; two are taken without replacement.
(a) Find P(exactly one red).
(b) Find P(at least one red).

(a) P(red then blue) + P(blue then red) = $2 \times \frac{8}{13} \times \frac{5}{12} = \frac{20}{39}$

(b) 1 − P(both blue) = $1 - \frac{5}{13} \times \frac{4}{12} = 1 - \frac{5}{39} = \frac{34}{39}$

Answer: (a) $\frac{20}{39}$ (b) $\frac{34}{39}$

8 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $3\sin^2 x + 8\cos x - 7 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2 x = 1 - \cos^2 x$: $8\cos x - 3\cos^2 x - 4 = 0$, i.e. $3\cos^2 x + 4 - 8\cos x = 0$

$(3\cos x - 2)(\cos x - 2) = 0$, so $\cos x = \frac{2}{3}$ or 2

$\cos x = \frac{2}{3}$: x = 48.2°, 311.8°

cos x = 2 has no solutions
Answer: x = 48.2°, 311.8°

9 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 4x + 3y = k is a tangent to the circle
$(x + 4)^2 + y^2 = 25$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (4, 3).
A unit step in this direction is (4, 3)/5, so the points of contact are
(−4, 0) ± 5 × (4, 3)/5 = (−4, 0) ± (4, 3)
T = (0, 3) or (−8, −3), and k = 4x + 3y gives:
k = 9 or k = −41
Answer: k = 9 or k = −41

10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $0 + 1\lambda = 0 + 1\mu$

j: $3 + (-1)\lambda = 0 + 2\mu$

Solving i and j: $\lambda = 1$, $\mu = 1$

k check: $-1 + 1(1) = 0$ and $-2 + 2(1) = 0$, equal

Point: $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{0} \end{pmatrix}$