Day 304 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^4 \frac{1}{(3x+1)}\,dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{1}{(3x+1)}\,dx = \frac{\ln(3x+1)}{3}$

$= [\frac{\ln(3x+1)}{3}]_0^4$

$= \frac{\ln 13}{3}$

Answer: $\frac{\ln 13}{3}$

2 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 24 students.

Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 24$, $\Sigma fx = 57$, $\Sigma fx^2 = 161$

Mean $= \frac{57}{24} = 2.38$

Variance $= \frac{161}{24} - (2.3750)^2 = 1.068$

Standard deviation = 1.03

Answer: mean 2.38, standard deviation 1.03

3 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{4}} \sin^2 2x\,dx$.

Use $\sin^2 2x = \frac{1}{2}(1 - \cos 4x)$

$\int \sin^2 2x\,dx = \frac{x}{2} - \frac{\sin 4x}{8}$

$= [\frac{x}{2} - \frac{\sin 4x}{8}]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{8}$

Answer: $\frac{\pi}{8}$

4 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(7\mathbf{i} + 3\mathbf{j})$ N and $(\mathbf{i} + 3\mathbf{j})$ N act on a particle of mass 4 kg.

Find the magnitude of the acceleration of the particle.

Resultant $= (7\mathbf{i} + 3\mathbf{j}) + (\mathbf{i} + 3\mathbf{j}) = 8\mathbf{i} + 6\mathbf{j}$

$|R| = \sqrt{(8^2 + 6^2)} = 10$ N

$a = F/m = 10 \div 4$

Answer: 2.5 m s^{-2}

5 Poisson distribution (CIE)

X ~ Po(1.5), modelling the number of flaws per metre of cloth.

Find, to 3 significant figures,

(a) P(X = 3)

(b) P(X ≤ 3)

(a) $P(X = 3) = e^{-1.5} \times \frac{1.5^3}{3!} = 0.126$

(b) $P(X \leq 3) = P(X = 0) + ... + P(X = 3) = 0.934$

Answer: (a) 0.126 (b) 0.934

6 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^3 - x^2 - 4x + 3$.
C is reflected in the x-axis.
Find its new equation.

Reflection in the x-axis: replace y by $-y$.
When reflected in the x-axis: $y = -x^3 + x^2 + 4x - 3$
Answer: $y = -x^3 + x^2 + 4x - 3$

7 Laws of logarithms: solving equations (All boards)

Solve $2 \ln x = \ln(x + 20)$.

$\ln x^2 = \ln(x + 20)$, so $x^2 = x + 20$
$x^2 - x - 20 = 0$, so $(x - 5)(x + 4) = 0$
$x > 0$ for $\ln x$ to exist, so $x = 5$
Answer: $x = 5$

8 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 4 and common ratio 4.
Find the smallest value of n for which the sum of the first n terms exceeds 10000.

$S_n = \frac{4(4^n - 1)}{3} > 10000$
$4^n > 7501$
$n > \frac{\lg 7501}{\lg 4} = 6.436$
So the smallest integer n is 7
Answer: $n = 7$

9 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 4 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$.

The coefficient of friction is 0.125. Take $g = 9.8$ m s^{-2}. Find the acceleration down the plane, and the distance moved in 3 s.

$\cos \alpha = 4/5$, $\sin \alpha = 3/5$
$R = mg \cos \alpha$, $F = \mu R$
$ma = mg \sin \alpha - \mu mg \cos \alpha$, so $a = 9.8(3/5 - 0.125 \times 4/5)$
$s = \frac{1}{2}at^2 = \frac{1}{2} \times 4.9 \times 3^2$
Answer: 4.9 m s^{-2}; 22.05 m

10 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x - 4y = k$ is a tangent to the circle
$(x - 4)^2 + (y - 4)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(3, -4)$.
A unit step in this direction is $(3, -4)/5$, so the points of contact are
$(4, 4) \pm 10 \times (3, -4)/5 = (4, 4) \pm (6, -8)$
T = $(10, -4)$ or $(-2, 12)$, and $k = 3x - 4y$ gives:
$k = 46$ or $k = -54$
Answer: $k = 46$ or $k = -54$