**Day 303** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 6 ml. Set mean 120 ml; 50 bottles have mean 121.7 ml.
Test at the 5% level whether the true mean is greater than 120.

$H_0$: $\mu = 120$, $H_1$: $\mu > 120$. Under $H_0$, $\overline{X} \sim N(120, \frac{6^2}{50})$

$z = \frac{121.7 - 120}{6/\sqrt{50}} = 2.003$

$P(Z > 2.003) = 0.0226 < 0.05$

Reject $H_0$: there is sufficient evidence that the mean has increased

**Answer: reject $H_0$: sufficient evidence of an increase in the mean**

## 2 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix}5\\5\\0\end{pmatrix}$ and $\begin{pmatrix}1\\1\\7\end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix}-4\\-4\\7\end{pmatrix}$

$|\text{AB}| = \sqrt{4^2 + 4^2 + 7^2} = \sqrt{81} = 9$

**Answer: AB = $\begin{pmatrix}-4\\-4\\7\end{pmatrix}$, distance 9**

## 3 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A puck of mass 8 kg is moving on a smooth horizontal surface at 1 m $s^{-1}$ when a constant horizontal force of 12 N, in the direction of motion, acts for 7 s.
Find the speed of the puck at the end of this time and the distance it moves.

$F = ma$: $a = 12 \div 8 = 1.5$

$v = u + at = 1 + 1.5 \times 7$

$s = ut + \frac{1}{2}at^2$

**Answer: 11.5 m $s^{-1}$; 43.75 m**

## 4 Chain rule (All boards)

Given that $y = (3x^2 + 2)^4$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = 0$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = 24x(3x^2 + 2)^3$

When $x = 0$: gradient = 0

**Answer: $\frac{dy}{dx} = 24x(3x^2 + 2)^3$; gradient 0**

## 5 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg lies on a rough horizontal table (coefficient of friction 0.2). It is joined by a light string over a smooth pulley to a particle of mass 7 kg hanging freely.
Take g = 9.8 m $s^{-2}$.
The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.2 \times 5g = 5a$

Hanging: $7g - T = 7a$

Add: $7g - 1g = 12a$

$a = 4.9$

$T = 7(9.8 - 4.9)$

**Answer: a = 4.9 m $s^{-2}$; T = 34.3 N**

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## 6 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 3 and sum to infinity 12.
Find the common ratio and the value of the 5th term.

$S_\infty = \frac{a}{1-r}$: $12 = \frac{3}{1-r}$

$1 - r = \frac{3}{12}$, so $r = \frac{3}{4}$

$u_5 = ar^4 = 3 \times \left(\frac{3}{4}\right)^4 = \frac{243}{256}$

**Answer: $r = \frac{3}{4}$, $u_5 = \frac{243}{256}$**

## 7 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 20, frequency 18, is a bar 4 cm wide, 1.125 cm tall.
(a) Find the bar height for width 20, frequency 20.
(b) Find the frequency of a 1.5 cm by 4 cm bar.

Area of bar $\propto$ frequency: $4 \times 1.125 = 4.5$ cm$^2$ represents 18
so 1 unit of frequency = 0.25 cm$^2$
(a) Width = 4 cm, area = $0.25 \times 20 = 5$ cm$^2$, height = $5 \div 4 = 1.25$ cm
(b) Area = $1.5 \times 4 = 6$ cm$^2$, frequency = $6 \div 0.25 = 24$
**Answer: (a) 1.25 cm (b) 24**

## 8 Intersection of two lines in 3D (CIE)

Line $l_1$: $\mathbf{r} = \begin{pmatrix} 5 \\ 5 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$.

Line $l_2$: $\mathbf{r} = \begin{pmatrix} 3 \\ 7 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}$.

Show that $l_1$ and $l_2$ intersect and find the point.

i: $5 + (-2)\lambda = 3 + (-1)\mu$
j: $5 + (-1)\lambda = 7 + (-2)\mu$
Solving i and j: $\lambda = 2$, $\mu = 2$
k check: $-9 + 3(2) = -3$ and $-1 + (-1)(2) = -3$, equal

Point: $\begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix}$

**Answer:** $\begin{pmatrix} \mathbf{1} \\ \mathbf{3} \\ \mathbf{-3} \end{pmatrix}$

## 9 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

x-intercepts at $x = -2$, 1 and 2, so $y = k(x + 2)(x - 1)(x - 2)$
The curve passes through $(0, -8)$: $k \times 2 \times (-1) \times (-2) = -8$
So $k = -2$
**Answer: $f(x) = -2(x + 2)(x - 1)(x - 2)$**

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## 10 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A salary starts at £20000 in year 1 and increases by 5% each year.
Find the total salary earned in the first 8 years, to the nearest pound.

Geometric series with a = 20000, r = 1.05

$S_8 = \frac{20000(1.05^8 - 1)}{0.05}$

= £190,982

**Answer: £190,982**