Day 302 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = e^{2x} - 6e^x$, and determine its nature.

$\frac{dy}{dx} = 2(e^x - 3)e^x = 0$

$x = \ln 3$

$\frac{d^2y}{dx^2} = 2(2e^x - 3)e^x$

At $x = \ln 3$: $\frac{d^2y}{dx^2} = 18 > 0$, so a minimum

Answer: $(\ln 3, -9)$ minimum

2 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A remote-control car moves with constant velocity $(5\mathbf{i} + 12\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north. Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed $= \sqrt{(5^2 + 12^2)} = 13$

$\tan\theta = 12 \div 5$

Answer: 13 m s^{-1}; 67.4°

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "$n^2 - n + 11$ is prime for every positive integer n."
Find a counter-example to show that the statement is false.

Test n = 1, 2, 3, ... in turn: all give primes until n = 11.

$n^2 - n + 11 = 121 - 11 + 11 = 121$

$121 = 11 \times 11$, which is not prime.

Answer: n = 11 (value 121)

4 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + 3y^2 - xy - 4x - y = 22$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at $(-3, -1)$.

Differentiate with respect to x: $2x + 6y\frac{dy}{dx} - x\frac{dy}{dx} - y - \frac{dy}{dx} - 4 = 0$

$\frac{dy}{dx} = \frac{2x - y - 4}{x + 1 - 6y}$

At $(-3, -1)$: gradient $= -\frac{9}{4}$

Answer: $\frac{dy}{dx} = \frac{2x - y - 4}{x + 1 - 6y}$; gradient $-\frac{9}{4}$

5 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $8^{-2x-2} = 4^{x-3}$.

Write both sides as powers of 2: $2^{3(-2x-2)} = 2^{2(x-3)}$

Equate the indices: $-6x - 6 = 2x - 6$

$-8x = 0$

$x = 0$

Answer: x = 0

6 Two independent binomials (All boards)

X ~ B(7, 0.3) and Y ~ B(6, 0.1) are independent.
Find P(X = 2 and Y ≤ 1), to 3 significant figures.

P(X = 2) = 0.3177

P(Y ≤ 1) = 0.8857

Independent, so multiply: 0.3177 × 0.8857 = 0.281

Answer: 0.281

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7 Power, driving force and resistance (CIE)

A car of mass 800 kg moves along a horizontal road. The engine works at a constant rate of 20 kW. The resistance to motion is constant at 500 N.

Find the acceleration when the speed is 8 m s^{-1}.

$P = Fv$: $F = 20000 \div 8 = 2500$ N

$F - R = ma$: $2500 - 500 = 800a$

Answer: 2.5 m s^{-2}

8 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{3x + 11}{(x + 1)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^3 \frac{3x + 11}{(x + 1)(x + 5)}\,dx$,

giving the answer as a single logarithm.

(a) $\frac{3x + 11}{(x + 1)(x + 5)} \equiv \frac{A}{x + 1} + \frac{B}{x + 5}$, so $3x + 11 \equiv A(x + 5) + B(x + 1)$

$x = -1$: $A = 2$; $x = -5$: $B = 1$

(b) $\int_0^3 \left(\frac{2}{x + 1} + \frac{1}{x + 5}\right) dx = [2\ln(x + 1) + \ln(x + 5)]_0^3$

$= 2\ln\frac{4}{1} + \ln\frac{8}{5} = \ln\left(\frac{128}{5}\right)$

Answer: (a) A = 2, B = 1 (b) $\ln\left(\frac{128}{5}\right)$

9 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 2 and common ratio 4.

Find the smallest value of n for which the sum of the first n terms exceeds 5000.

$S_n = \frac{2(4^n - 1)}{3} > 5000$

$4^n > 7501$

$n > \frac{\lg 7501}{\lg 4} = 6.436$

So the smallest integer n is 7

Answer: n = 7

10 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.

$P(X < 18) = 0.1587$ and $P(X > 54) = 0.0228$.

Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.999$

$18 = \mu + (-0.9998)\sigma$ and $54 = \mu + 1.999\sigma$

Subtract: $36 = 2.999\,\sigma$, so $\sigma = 12.0$

$\mu = 18 - (-0.9998)(12.00) = 30.0$

Answer: μ = 30.0, σ = 12.0