Day 301 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 6 \sin 5x \, dx$,

(b) the exact value of $\int_0^{\frac{\pi}{10}} 6 \sin 5x \, dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $-\frac{6 \cos 5x}{5} + c$

(b) $[-\frac{6 \cos 5x}{5}]_0^{\frac{\pi}{10}}$

$= \frac{6}{5}$

Answer: (a) $-\frac{6 \cos 5x}{5} + c$ (b) $\frac{6}{5}$

2 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (3, −5) and B is the point (6, 4).

Find the perpendicular bisector of AB as ax + by + c = 0, with integer a, b, c.

Midpoint of AB = $(\frac{9}{2}, -\frac{1}{2})$

Gradient of AB = $\frac{9}{3} = 3$

Perpendicular gradient = $-\frac{1}{3}$

$y + \frac{1}{2} = -\frac{1}{3}(x - \frac{9}{2})$, which gives $x + 3y - 3 = 0$

Answer: $x + 3y - 3 = 0$

3 Normal distribution probabilities (All boards)

The length X cm of a metal rod is normal with mean 100, s.d. 3.

Find, to 3 s.f., the probability a value is

(a) less than 104.5,

(b) between 104.5 and 106.

$X \sim N(100, 3^2)$

(a) $P(X < 104.5) = 0.933$

(b) $P(104.5 < X < 106) = 0.9772 - 0.9332 = 0.0441$

Answer: (a) 0.933 (b) 0.0441

4 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = k u_n + 2$, $u_1 = 2$.

Given that $u_3 = 2$, find the possible values of k.

$u_2 = 2k + 2$

$u_3 = k(2k + 2) + 2 = 2k^2 + 2k + 2$

$2k^2 + 2k + 2 = 2$, so $2k^2 + 2k = 0$

$2k(k + 1) = 0$

$k = -1$ or $k = 0$

Answer: $k = -1$ or $k = 0$

5 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (6 kg) and B (10 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 2.5 m above the ground, then released.

Take $g = 9.8$ m s^{-2}.

Find the speed with which B hits the ground.

$a = (10 - 6)g \div (10 + 6) = 2.45$

$v^2 = 0 + 2 \times 2.45 \times 2.5 = 12.25$

$v = 3.5$

Answer: 3.5 m s^{-1}

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6 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $24\cos\theta + 7\sin\theta \equiv 25\cos(\theta - 16.3°)$,

find the greatest value of $\frac{1}{(28 + 24\cos\theta + 7\sin\theta)}$.

Denominator is smallest when $25\cos(\theta - 16.3°) = -25$

Smallest denominator $= 28 - 25 = 3$

Greatest value $= \frac{1}{3}$

Answer: $\mathbf{\frac{1}{3}}$

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 8 ml. Set mean 250 ml; 36 bottles have mean 252.7 ml.
Test at the 5% level whether the true mean is greater than 250.

$H_0: \mu = 250$, $H_1: \mu > 250$. Under H_0, $\overline{X} \sim N(250, \frac{8^2}{36})$

$z = \frac{252.7 - 250}{8/\sqrt{36}} = 2.025$

$P(Z > 2.025) = 0.0214 < 0.05$

Reject H_0: there is sufficient evidence that the mean has increased

Answer: reject H_0: sufficient evidence of an increase in the mean

8 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $2x^2y + 2xy + 3y^3 = -81$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at $(0, -3)$.

Differentiate with respect to x: $2x^2\frac{dy}{dx} + 4xy + 2x\frac{dy}{dx} + 9y^2\frac{dy}{dx} + 2y = 0$

$\frac{dy}{dx} = \frac{2y(-2x - 1)}{2x^2 + 2x + 9y^2}$

At $(0, -3)$: gradient $= \frac{2}{27}$

Answer: $\mathbf{\frac{dy}{dx} = \frac{2y(-2x - 1)}{2x^2 + 2x + 9y^2}}$**; gradient** $\mathbf{\frac{2}{27}}$

9 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(7, 2) and B(5, 8), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:

$49 + (h - 2)^2 = 25 + (h - 8)^2$

Expanding and cancelling h^2: $-4h + 53 = -16h + 89$

$h = 3$

$r^2 = CA^2 = 50$

Answer: centre (0, 3), $x^2 + (y - 3)^2 = 50$

10 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 10 s, (b) the distance between the stations.

(a) a = gradient = 16 ÷ 10

(b) Distance = area of trapezium = ½ × (44 + 30) × 16

Answer: (a) 1.6 m s^{-2} (b) 592 m