Day 300 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moves in a straight line with constant acceleration. Its initial speed is 13 m s^{-1}, its final speed is 25 m s^{-1} and its acceleration is 4 m s^{-2}.
Find the time taken.

$v = u + at$
$25 = 13 + 4t$
$4t = 12$
Answer: 3 s

2 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = 1$ and θ is acute,
find the exact value of $\sec\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = 1$, so $\sec^2\theta = 1 + 1 = 2$
$\sec\theta = \sqrt{2}$ (positive as θ is acute)
Answer: $\sqrt{2}$

3 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^2 - 9x > -9$.

Rearrange: $2x^2 - 9x + 9 > 0$
Factorise: $(x - 3)(2x - 3) > 0$
Critical values: $x = \frac{3}{2}$ and $x = 3$
Sketch the ∪-shaped parabola: $x < \frac{3}{2}$ or $x > 3$
Answer: $x < \frac{3}{2}$ or $x > 3$

4 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 70 kg stands on weighing scales in a lift. The lift decelerates at 0.25 m s^{-2} while moving downwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is upwards, magnitude 0.25
Taking up as positive: $R - 70 \times 9.8 = 70 \times 0.25$
$R = 70(9.8 + 0.25)$
Answer: 703.5 N

5 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 1.5 cm per second.
Find the rate at which its area is increasing when r = 8 cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$
$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 1.5$
When r = 8: $\frac{dA}{dt} = 24\pi$
Answer: 24π cm^2/s

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6 Binomial critical region (All boards)

$X \sim B(40, p)$. H_0: $p = 0.3$ is to be tested against H_1: $p > 0.3$ at the 10% significance level.
Find the critical region and the actual significance level.

Under H_0, $X \sim B(40, 0.3)$
$P(X \geq 17) = 0.0633 \leq 0.1$
$P(X \geq 16) = 0.1151 > 0.1$
The critical region is $X \geq 17$
Answer: $X \geq 17$; actual significance level 6.33%

7 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1 - u_n}$, $u_1 = \frac{1}{2}$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{20} and $\sum_{r=1}^{20} u_r$.

(a) $u_2 = 2$, $u_3 = -1$, $u_4 = \frac{1}{2}$
$u_4 = u_1$ so the sequence repeats with period 3
(b) $20 = 3 \times 6 + 2$, so $u_{20} = u_2 = 2$
Sum of one cycle $= \frac{3}{2}$, so the sum $= 6 \times \frac{3}{2} + \frac{5}{2} = \frac{23}{2}$
Answer: period 3; $u_{20} = 2$; sum $= \frac{23}{2}$

8 Normal approximation to the binomial (AQA, OCR, CIE)

$X \sim B(60, 0.3)$. Use a normal approximation to find $P(19 \leq X \leq 26)$, to 3 significant figures.

$Y \sim N(np, np(1 - p)) = N(18.00, 12.60)$, so $\sigma = \sqrt{12.60} = 3.550$
Use a continuity correction.
$P(19 \leq X \leq 26) \approx P(18.5 < Y < 26.5) = 0.436$
Answer: 0.436

9 Simplify using double angle formulae (All boards)

Simplify $\operatorname{cosec} 2x + \cot 2x$,
giving your answer as a single trigonometric term.

$\operatorname{cosec} 2x + \cot 2x = \frac{1 + \cos 2x}{\sin 2x}$

$= \frac{2 \cos^2 x}{2 \sin x \cos x} = \cot x$

Answer: cot x

10 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, a quadratic.
(a) Find the coordinates of the local maximum point of $y = |f(x)|$.
(b) How many solutions has $|f(x)| = 1$?

(a) $y = |f(x)|$ reflects the part of the curve below the x-axis in the x-axis:
$(0, -1)$ becomes $(0, 1)$
(b) The maximum of the middle part is 1. Compare $k = 1$ with 1:
$k = 1$ gives 3 intersections with $y = k$.
Answer: (a) (0, 1) (b) 3