Day 30 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A company employs 160 managers, 40 technicians and 60 operators.
A stratified sample of size 26 is taken. Find the number selected from each group.

Total = 160 + 40 + 60 = 260

Sampling fraction = 26/260 = $\frac{1}{10}$

managers: 160 ÷ 10 = 16
technicians: 40 ÷ 10 = 4
operators: 60 ÷ 10 = 6
Answer: managers 16, technicians 4, operators 6

2 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 6\sec^2 4x\,dx$,

(b) the exact value of $\int_0^{\frac{\pi}{16}} 6\sec^2 4x\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{3\tan 4x}{2} + c$

(b) $\left[\frac{3\tan 4x}{2}\right]_0^{\frac{\pi}{16}}$

$= \frac{3}{2}$

Answer: (a) $\frac{3\tan 4x}{2} + c$ (b) $\frac{3}{2}$

3 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg is raised and lowered by a vertical cable. The lift accelerates downwards at 0.5 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolve vertically (up positive): $T - 300 \times 9.8 = -300 \times 0.5$
$T = 300(9.8 - 0.5)$
Answer: 2790 N

4 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{6}{3+\sqrt{7}}$ in the form $p + q\sqrt{7}$, where p and q are integers.

Multiply top and bottom by the conjugate $(3 - \sqrt{7})$:
Denominator: $(3+\sqrt{7})(3-\sqrt{7}) = 3^2 - 1^2 \times 7 = 2$
Numerator: $(6)(3-\sqrt{7}) = 18 - 6\sqrt{7}$
So the fraction = $\frac{18-6\sqrt{7}}{2} = 9 - 3\sqrt{7}$
Answer: $9 - 3\sqrt{7}$

5 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that its position vector relative to O at time t s is $\mathbf{r} = (2t^2 - 2t)\mathbf{i} + (t^3)\mathbf{j}$ metres.
Find the velocity when t = 4, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (4t - 2)\mathbf{i} + (3t^2)\mathbf{j}$
t = 4: $\mathbf{v} = (14)\mathbf{i} + (48)\mathbf{j}$
Speed = $\sqrt{(14^2 + 48^2)}$
Answer: $\mathbf{v} = (14)\mathbf{i} + (48)\mathbf{j}$ m s^{-1}; speed 50 m s^{-1}

6 Two independent binomials (All boards)

X ~ B(11, 0.2) and Y ~ B(9, 0.1) are independent.
Find P(X = 1 and Y ≤ 2), to 3 significant figures.

P(X = 1) = 0.2362
P(Y ≤ 2) = 0.9470
Independent, so multiply: 0.2362 × 0.9470 = 0.224
Answer: 0.224

7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{x + 10}{(x + 1)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{x + 10}{(x + 1)(x + 4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{x + 10}{(x + 1)(x + 4)} \equiv \frac{A}{x + 1} + \frac{B}{x + 4}$, so $x + 10 \equiv A(x + 4) + B(x + 1)$
$x = -1$: $A = 3$; $x = -4$: $B = -2$

(b) $\int_0^4 \left(\frac{3}{x + 1} + \frac{-2}{x + 4}\right) dx = [3 \ln(x + 1) - 2 \ln(x + 4)]_0^4$

$= 3 \ln \frac{5}{1} - 2 \ln \frac{8}{4} = \ln\left(\frac{125}{4}\right)$

Answer: (a) $A = 3$, $B = -2$ (b) $\ln\left(\frac{125}{4}\right)$

8 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(−5, −3) and B(1, −5), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:
$25 + (h + 3)^2 = 1 + (h + 5)^2$
Expanding and cancelling h^2: $6h + 34 = 10h + 26$
$h = 2$
$r^2 = CA^2 = 50$
Answer: centre (0, 2), $x^2 + (y - 2)^2 = 50$

9 Quotient rule (All boards)

Find the x-coordinate(s) of the stationary point(s) of the curve $y = \frac{x^2}{x - 4}$.

$\frac{dy}{dx} = \frac{x(x - 8)}{(x - 4)^2}$

Stationary points where $\frac{dy}{dx} = 0$, i.e. the numerator is zero

Numerator: $x(x - 8) = 0$
$x = 0$ or $x = 8$
Answer: $x = 0$, $x = 8$

10 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x + 4)(x + 2)(x - 2) \leq 0$.

Critical values: $x = -4$, $x = -2$, $x = 2$
The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.
Reading from the sketch: $x \leq -4$ or $-2 \leq x \leq 2$
Answer: $x \leq -4$ or $-2 \leq x \leq 2$