**Day 3** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations
$x = 4 + 4\cos t$, $y = 4\sin t$, for $0 \leq t < 2\pi$. Find a Cartesian equation of the curve.

$\cos t = \frac{x - 4}{4}$ and $\sin t = \frac{y}{4}$

Use $\cos^2 t + \sin^2 t = 1$:

$(x - 4)^2 + y^2 = 16$

**Answer: $(x - 4)^2 + y^2 = 16$**

## 2 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A boat moves with constant velocity $(-10\mathbf{i} + 24\mathbf{j})$ m s$^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north.
Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed $= \sqrt{(-10^2 + 24^2)} = 26$

$\tan\theta = 24 \div 10$

**Answer: 26 m s$^{-1}$; 67.4°**

## 3 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = \text{cosec } x$ with respect to $x$,

and find the exact gradient when $x = \frac{\pi}{4}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = -\cot x \text{ cosec } x$

At $x = \frac{\pi}{4}$: $\frac{dy}{dx} = -\sqrt{2}$

**Answer: $\frac{dy}{dx} = -\cot x \text{ cosec } x$; gradient $-\sqrt{2}$**

## 4 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are $(-7, 4)$, $(-3, 2)$ and $(1, 10)$.
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB $= \frac{2}{-4} = -\frac{1}{2}$

Gradient BC $= \frac{8}{4} = 2$

$(-\frac{1}{2}) \times 2 = -1$, so AB and BC are perpendicular.

(b) AB $= 2\sqrt{5}$ and BC $= 4\sqrt{5}$

Area $= \frac{1}{2} \times$ AB $\times$ BC $= 20$

**Answer: (b) area = 20**

## 5 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: mean temperature (°C) at Heathrow has mean 16, sd 2.
Outliers are more than 2 sd from the mean.
Are the values 21 and 15 outliers?

Limits: $16 \pm 2 \times 2$, so from 12 to 20

21 is outside the limits, so it is an outlier

15 is inside the limits, so it is not an outlier

**Answer: 21: outlier; 15: not an outlier**

## 6 Smooth inclined plane (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block slides from rest down a line of greatest slope of a smooth plane inclined at $\alpha$, where $\sin\alpha = \frac{3}{5}$ to the horizontal.

Take $g = 9.8$ m s$^{-2}$.

Find its acceleration, and the distance it travels in 5 s.

Along the plane: $mg\sin\alpha = ma$, so $a = 9.8 \times 0.6 = 5.88$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 5.88 \times 5^2$

**Answer: 5.88 m s$^{-2}$; 73.5 m**

## 7 Intersection of two lines in 3D (CIE)

Line $l_1$: $\mathbf{r} = \begin{pmatrix}7\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}3\\2\\0\end{pmatrix}$.

Line $l_2$: $\mathbf{r} = \begin{pmatrix}4\\-2\\2\end{pmatrix} + \mu\begin{pmatrix}3\\1\\-2\end{pmatrix}$.

Show that $l_1$ and $l_2$ intersect and find the point.

i: $7 + 3\lambda = 4 + 3\mu$

j: $0 + 2\lambda = -2 + 1\mu$

Solving i and j: $\lambda = -1$, $\mu = 0$

k check: $2 + 0(-1) = 2$ and $2 + (-2)(0) = 2$, equal

Point: $\begin{pmatrix}4\\-2\\2\end{pmatrix}$

**Answer:** $\begin{pmatrix}\mathbf{4}\\\mathbf{-2}\\\mathbf{2}\end{pmatrix}$

## 8 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass, X g, of a loaf is normally distributed with mean 20 and s.d. 6.

A random sample of 16 is taken. Find the probability that the mean mass is less than 17.75 (3 s.f.).

$\overline{X} \sim N(20, \frac{6^2}{16})$, standard deviation 1.500

$z = \frac{17.75 - 20}{1.500} = -1.500$

$P = 0.0668$

**Answer: 0.0668**

## 9 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point $(-5, -2)$ and B is the point $(-3, -6)$.

Find the perpendicular bisector of AB as $ax + by + c = 0$, with integer a, b, c.

Midpoint of AB = $(-4, -4)$

Gradient of AB = $\frac{-4}{2} = -2$

Perpendicular gradient = $\frac{1}{2}$

$y + 4 = \frac{1}{2}(x + 4)$, which gives $x - 2y - 4 = 0$

**Answer: $x - 2y - 4 = 0$**

## 10 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\tan^2 x + \sec x - 1 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x + \sec x - 3 = 0$

$(\sec x - 1)(2\sec x + 3) = 0$, so $\sec x = 1$ or $-\frac{3}{2}$

$\sec x = 1$, so $\cos x = 1$: $x =$

$\sec x = -\frac{3}{2}$, so $\cos x = -\frac{2}{3}$: $x = 131.8°, 228.2°$

**Answer: x = 131.8°, 228.2°**