Day 299 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} 4 \\ 12 \\ 3 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{4^2 + 12^2 + 3^2} = \sqrt{169} = 13$

Unit vector $= \frac{1}{13}\mathbf{a}$

Answer: $|\mathbf{a}| = 13$, unit vector $\frac{1}{13}\begin{pmatrix} 4 \\ 12 \\ 3 \end{pmatrix}$

2 Probability without replacement (All boards)

A bag has 5 red and 7 blue counters; two are taken without replacement.

(a) Find P(exactly one red).

(b) Find P(at least one red).

(a) P(red then blue) + P(blue then red) $= 2 \times \frac{5}{12} \times \frac{7}{11} = \frac{35}{66}$

(b) $1 - $ P(both blue) $= 1 - \frac{7}{12} \times \frac{6}{11} = 1 - \frac{7}{22} = \frac{15}{22}$

Answer: (a) $\frac{35}{66}$ (b) $\frac{15}{22}$

3 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 5 strips to estimate

$$\int_0^1 \frac{1}{\sqrt{1 + x^3}}\,dx,$$

giving your answer to 3 significant figures.

$h = \frac{1 - 0}{5} = 0.2$

x: 0, 0.2, 0.4, 0.6, 0.8, 1

y: 1.000, 0.9960, 0.9695, 0.9068, 0.8133, 0.7071

Area $\approx \frac{h}{2}[y_0 + y_5 + 2(y_1 + \ldots + y_4)]$

$= \frac{0.2}{2} \times [1.000 + 0.7071 + 2 \times 3.686] = 0.9078$

Answer: 0.908

4 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are $(-4, -5)$, $(-3, -1)$ and $(-7, 0)$.

(a) Show that angle ABC = 90°.

(b) Find the area of triangle ABC.

(a) Gradient AB $= \frac{-4}{-1} = 4$

Gradient BC $= \frac{1}{-4} = -\frac{1}{4}$

$4 \times (-\frac{1}{4}) = -1$, so AB and BC are perpendicular.

(b) AB $= \sqrt{17}$ and BC $= \sqrt{17}$

Area $= \frac{1}{2} \times$ AB $\times$ BC $= \frac{17}{2}$

Answer: (b) area $= \frac{17}{2}$

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5 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected horizontally at 156.8 m s^{-1} from a point high above the ground.
Take g = 9.8 m s^{-2}.
Find the speed and the direction of motion 12 s later, to 1 d.p. below the horizontal.

Vertical speed = 9.8 × 12 = 117.6
Speed = $\sqrt{(156.8^2 + 117.6^2)}$ = 196
tan α = 117.6 ÷ 156.8, α = 36.9°
Answer: 196 m s^{-1} at 36.9° below horizontal

6 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has second term 128 and fifth term 2.
Find (a) the common ratio, (b) the first term, (c) the sum to infinity.

$ar = 128$ and $ar^4 = 2$, so $r^3 = \frac{2}{128} = \frac{1}{64}$

$r = \frac{1}{4}$, and $a = 128 \div \frac{1}{4} = 512$

$S_\infty = \frac{a}{1 - r} = \frac{512}{1 - \frac{1}{4}} = \frac{2048}{3}$

Answer: (a) $r = \frac{1}{4}$ (b) a = 512 (c) $\frac{2048}{3}$

7 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 10, frequency 12, is a bar 5 cm wide, 4.8 cm tall.
(a) Find the bar height for width 30, frequency 12.
(b) Find the frequency of a 3 cm by 2 cm bar.

Area of bar ∝ frequency: 5 × 4.8 = 24 cm^2 represents 12
so 1 unit of frequency = 2 cm^2
(a) Width = 15 cm, area = 2 × 12 = 24 cm^2, height = 24 ÷ 15 = 1.6 cm
(b) Area = 3 × 2 = 6 cm^2, frequency = 6 ÷ 2 = 3
Answer: (a) 1.6 cm (b) 3

8 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = 7x − 11 meets the circle $(x + 2)^2 + y^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute y = 7x − 11: $(x + 2)^2 + (7x - 11)^2 = 25$
$x^2 - 3x + 2 = 0$
(x − 2)(x − 1) = 0, so x = 1 or x = 2
y = 7x − 11: A = (1, −4), B = (2, 3)
$AB^2 = 1^2 + 7^2 = 50$
Answer: A(1, −4), B(2, 3), AB = $5\sqrt{2}$

9 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $3 \sin^2 x - 2 \cos x - 2 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2 x = 1 - \cos^2 x$: $1 - 3 \cos^2 x - 2 \cos x = 0$, i.e. $3 \cos^2 x + 2 \cos x - 1 = 0$
$(3 \cos x - 1)(\cos x + 1) = 0$, so $\cos x = \frac{1}{3}$ or −1

$\cos x = \frac{1}{3}$: x = 70.5°, 289.5°

cos x = −1: x = 180.0°
Answer: x = 70.5°, 180.0°, 289.5°

10 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 4 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

The coefficient of friction is 0.125. Take $g = 9.8$ m s^{-2}. Find the acceleration down the plane, and the distance moved in 4 s.

$\cos\alpha = 4/5$, $\sin\alpha = 3/5$

$R = mg\cos\alpha$, $F = \mu R$

$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(3/5 - 0.125 \times 4/5)$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 4.9 \times 4^2$

Answer: 4.9 m s^{-2}; 39.2 m