Day 298 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of:
(a) y = f(x) − 1
(b) y = −f(x)

(a) y = f(x) − 1 is a translation $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

(b) y = −f(x) is a reflection in the x-axis.

Answer: (a) translation $\begin{pmatrix} \mathbf{0} \\ \mathbf{-1} \end{pmatrix}$ (b) reflection in the x-axis

2 Conditional probability from a count (All boards)

100 members: 42 swim, 44 cycle, 47 do neither.
(a) How many do both?
(b) Given a person who swims, find P(this person cycles).

(a) At least one: 100 − 47 = 53. Both = 42 + 44 − 53 = 33

(b) Of the 42 who swim, 33 also do the other: P = $\frac{33}{42} = \frac{11}{14}$

Answer: (a) 33 (b) $\frac{11}{14}$

3 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when
$-x^3 + x^2 - 3x - 10$ is divided by (x + 1).

$-x^2 \times (x + 1) = -x^3 - x^2$, leaving $2x^2 - 3x - 10$
$2x \times (x + 1) = 2x^2 + 2x$, leaving $-5x - 10$
$-5 \times (x + 1) = -5x - 5$, leaving -5
Quotient $-x^2 + 2x - 5$, remainder -5

Answer: quotient $-x^2 + 2x - 5$, remainder -5

4 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: maximum relative humidity (%) at Hurn has mean 91, sd 7.
Outliers are more than 2 sd from the mean.
Are the values 106 and 74 outliers?

Limits: 91 ± 2 × 7, so from 77 to 105
106 is outside the limits, so it is an outlier
74 is outside the limits, so it is an outlier
Answer: 106: outlier; 74: outlier

5 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate y = 5 cot 3x with respect to x,
and find the exact gradient when x = $\frac{\pi}{12}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = -15 \operatorname{cosec}^2 3x$
At x = $\frac{\pi}{12}$: $\frac{dy}{dx} = -30$

Answer: $\frac{dy}{dx} = -15 \operatorname{cosec}^2 3x$; gradient -30

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6 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(5, 0) and B(−5, −10), and its centre lies on the x-axis. Find the centre and the equation of the circle.

Let the centre be (h, 0). Then $CA^2 = CB^2$:

$(h - 5)^2 + 0 = (h + 5)^2 + 100$

Expanding and cancelling h^2: $-10h + 25 = 10h + 125$

$h = -5$

$r^2 = CA^2 = 100$

Answer: centre $(-5, 0)$, $(x + 5)^2 + y^2 = 100$

7 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 1 : 3, and N lies on OB with ON : NB = 3 : 2. Find OM and NM in terms of a and b.

$AB = b - a$, so $AM = \frac{1}{4}(b - a)$

$OM = a + AM = \frac{3a}{4} + \frac{b}{4}$

$ON = \frac{3b}{5}$, so $NM = OM - ON = \frac{3a}{4} - \frac{7b}{20}$

Answer: $OM = \frac{3a}{4} + \frac{b}{4}$, $NM = \frac{3a}{4} - \frac{7b}{20}$

8 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 4 kg lies on a rough horizontal table (coefficient of friction 0.5). It is joined by a light string over a smooth pulley to a particle of mass 3 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.5 \times 4g = 4a$

Hanging: $3g - T = 3a$

Add: $3g - 2g = 7a$

$a = 1.4$

$T = 3(9.8 - 1.4)$

Answer: $a = 1.4$ m s^{-2}; $T = 25.2$ N

9 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value of $\lim_{\theta \to 0} \frac{\tan 4\theta \sin 2\theta}{1 - \cos 4\theta}$.

$\tan 4\theta \sin 2\theta \approx 4\theta \times 2\theta = 8\theta^2$

$1 - \cos 4\theta \approx \frac{16\theta^2}{2}$

Ratio $= \frac{8\theta^2}{\frac{16\theta^2}{2}} = 1$

Answer: 1

10 Work-energy: speed after falling or sliding (CIE)

A particle of mass 4 kg slides from rest down a smooth curved slope, starting 10 m above the bottom.

Take $g = 9.8$ m s^{-2}.

Use energy to find its speed at the bottom of the slope.

Loss of PE = gain of KE: $m \times 9.8 \times 10 = \frac{1}{2} m v^2$

$v^2 = 19.6 \times 10 = 196$

$v = 14$

Answer: 14 m s^{-1}