Day 297 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A school has 80 students in Year 7, 56 in Year 8 and 80 in Year 9.
A stratified sample of size 27 is taken. Find the number selected from each group.

Total = 80 + 56 + 80 = 216

Sampling fraction = 27/216 = $\frac{1}{8}$

Year 7: 80 ÷ 8 = 10
Year 8: 56 ÷ 8 = 7
Year 9: 80 ÷ 8 = 10
Answer: Year 7 10, Year 8 7, Year 9 10

2 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x + 5| = |-x - 2|$.

Either $2x + 5 = -x - 2$ or $2x + 5 = -(-x - 2)$

$2x + 5 = -x - 2$ gives $x = -\frac{7}{3}$

$2x + 5 = -(-x - 2)$ gives $x = -3$

Answer: $x = -3$ and $x = -\frac{7}{3}$

3 Independent and mutually exclusive events (All boards)

A and B are mutually exclusive events with $P(A) = 0.25$ and $P(A \cup B) = 1$.
(a) Find P(B).
(b) Find $P(A' \cap B')$.

(a) Mutually exclusive: $P(A \cup B) = P(A) + P(B)$
$P(B) = 1 - 0.25 = 0.75$
(b) $P(A' \cap B') = 1 - P(A \cup B) = 0$
Answer: (a) 0.75 (b) 0

4 Trig equations in radians (All boards)

Solve $\tan 2x = 1.5$ for $0 \leq x \leq \pi$,
giving answers to 3 significant figures.

Let $\theta = 2x$, so $\tan \theta = 1.5$ for $0 \leq \theta \leq 2\pi$
Calculator: $\theta = 0.9828$, then use the symmetry of the graph
$\theta = 0.9828, 4.124$
$x = \frac{\theta}{2}$
$x = 0.4914, 2.062$
Answer: x = 0.491, 2.06

5 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(n + 5)^2 - (n - 5)^2$ is a multiple of 20 for all integer values of n.

$(n + 5)^2 = n^2 + 10n + 25$
$(n - 5)^2 = n^2 - 10n + 25$
Difference = $20n = 20 \times n$
n is an integer, so this is a multiple of 20 for every integer n.
Answer: difference = 20n, a multiple of 20

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6 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $8\cos\theta + 15\sin\theta \equiv 17\cos(\theta - 61.9°)$,
solve $8\cos\theta + 15\sin\theta = -3.5$ for $0 \le \theta \le 360°$. (1 d.p.)

$17\cos(\theta - 61.9°) = -3.5$, so $\cos(\theta - 61.9°) = \frac{-3.5}{17}$

$\theta - 61.9° = \pm 101.9°$
$\theta = 61.9° + 101.9° = 163.8°$
$\theta = 61.9° - 101.9° = -40.0°$, so $\theta = 320.0°$
Answer: $\theta = 163.8°, 320.0°$

7 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $x^2 - 8x + 20 > 0$ for all real values of x.

Complete the square: $x^2 - 8x + 20 = (x - 4)^2 + 4$
$(x - 4)^2 \ge 0$ for all real x, so $(x - 4)^2 \ge 0$.
Adding 4: the expression is ≥ 4, so > 0 for all real x.
Answer: $x^2 - 8x + 20 > 0$ for all x

8 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = 2\sqrt{x}$, the x-axis
and the lines $x = 0$ and $x = 4$.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^4 y^2\,dx = \pi \int_0^4 4x\,dx$
$= \pi[2x^2]_0^4$
$= 32\pi$
Answer: 32π

9 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 10 kg is held on a rough plane ($\tan\alpha = \frac{4}{3}$, $\mu = 0.5$) by a force P N up the plane, as shown.

Take $g = 9.8$ m s^{-2}. The block is about to move up the plane. Find P.

$R = mg\cos\alpha = 98 \times 3/5 = 58.8$
Friction (limiting) $F = \mu R = 29.4$
Moving up, friction acts down: $P = mg\sin\alpha + F$
$mg\sin\alpha = 78.4$
Answer: 107.8 N

10 Overtaking problem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A police car starts from rest and accelerates at 4 m s^{-2}. At that moment it is passed by a speeding van travelling at constant speed 16 m s^{-1}.
Find how long it takes to catch up, and the distance travelled.

Same distance: $\frac{1}{2} \times 4 \times t^2 = 16\,t$
$t(2\,t - 16) = 0$, $t \ne 0$ so $t = 16 \div 2$
Distance $= 16 \times 8$
Answer: 8 s, 128 m