Day 296 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 24 and the coefficient of x^2 is 240.
Find the values of n and k.

$nk = 24$, so $k = \frac{24}{n}$

$\frac{n(n-1)}{2}k^2 = 240$, so $\frac{n(n-1)}{2} \times \frac{576}{n^2} = 240$

$\frac{(n-1) \times 576}{2n} = 240$, so $576(n-1) = 480n$

$n = 6$, and $k = 4$

Answer: n = 6, k = 4

2 Trig equations in radians (All boards)

Solve $\tan x = 3.5$ for $0 \leq x \leq 2\pi$,
giving answers to 3 significant figures.

$\tan \theta = 3.5$ for $0 \leq x \leq 2\pi$
Calculator: $x = 1.292$, then use the symmetry of the graph
$x = 1.292, 4.434$
Answer: x = 1.29, 4.43

3 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has $E(X) = 6$ and $Var(X) = 2$.
Find (a) $E(3X + 5)$,
(b) $Var(3X + 5)$, (c) $E(X^2)$.

(a) $E(aX + b) = aE(X) + b = 3 \times 6 + 5 = 23$

(b) $Var(aX + b) = a^2Var(X) = 9 \times 2 = 18$

(c) $Var(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 2 + 36 = 38$

Answer: (a) 23 (b) 18 (c) 38

4 Car towing a trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 800 kg tows a trailer of mass 200 kg along a straight horizontal road by a light horizontal tow bar. The resistances are 300 N on the car and 100 N on the trailer.
The engine's driving force is 2000 N. Find the acceleration and the tension in the tow bar.

Whole system: $2000 - 300 - 100 = 1000\,a$
$a = 1.6$
Trailer: $T - 100 = 200 \times 1.6$

Answer: a = 1.6 m s^{-2}; T = 420 N

5 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(8 - x)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

Take out 8: $(8 - x)^{-\frac{1}{3}} = 8^{-\frac{1}{3}}(1 + -\frac{1}{8}x)^{-\frac{1}{3}} = \frac{1}{2}(1 + -\frac{1}{8}x)^{-\frac{1}{3}}$

Expand $(1 + -\frac{1}{8}x)^{-\frac{1}{3}} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Multiplying by $\frac{1}{2}$: coefficients of 1, x, x^2, x^3 are $\frac{1}{2}$, $\frac{1}{48}$, $\frac{1}{576}$, $\frac{7}{41472}$

Valid when $|\frac{1}{8}x| < 1$, that is $|x| < 8$

Answer: (a) $\frac{1}{2} + \frac{1}{48}x + \frac{1}{576}x^2 + \frac{7}{41472}x^3 + ...$ (b) $|x| < 8$

Day 296 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A javelin is projected from level ground with horizontal velocity component 40 m s^{-1} and vertical velocity component 39.2 m s^{-1} upwards.
Take g = 9.8 m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 39.2\,t - 4.9t^2$, so $t = 39.2 \div 4.9$
(b) Range = horizontal speed × time = 40 × 8
Answer: (a) 8 s (b) 320 m

7 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot^2 x - 5\operatorname{cosec} x + 7 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $\operatorname{cosec}^2 x - 5\operatorname{cosec} x + 6 = 0$
$(\operatorname{cosec} x - 3)(\operatorname{cosec} x - 2) = 0$, so $\operatorname{cosec} x = 3$ or 2
$\operatorname{cosec} x = 3$, so $\sin x = \frac{1}{3}$: x = 19.5°, 160.5°
$\operatorname{cosec} x = 2$, so $\sin x = \frac{1}{2}$: x = 30.0°, 150.0°
Answer: x = 19.5°, 30.0°, 150.0°, 160.5°

8 Binomial: smallest n (All boards)

Given that each lottery ticket wins a prize independently with probability 0.2, find the least number of tickets needed for the probability of at least one success to exceed 0.9.

P(at least one) = $1 - 0.8^n > 0.9$
$0.8^n < 0.1$
$n > \frac{\log 0.1}{\log 0.8} = 10.32$
Answer: n = 11

9 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{-5}{x+2} - 3$.
(a) Write down the asymptotes.
(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = -2$, and $y \to -3$ as $x \to \pm\infty$
Asymptotes: $x = -2$ and $y = -3$
(b) x = 0: $y = -\frac{5}{2} - 3 = -\frac{11}{2}$
y = 0: $\frac{-5}{x+2} = 3$, so $x + 2 = -\frac{5}{3}$, $x = -\frac{11}{3}$
Answer: (a) $x = -2$, $y = -3$ (b) $(0, -\frac{11}{2})$ and $(-\frac{11}{3}, 0)$

10 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 16\,e^x + 48 = 0$,
giving your answers as exact values.

Let $y = e^x$: $y^2 - 16\,y + 48 = 0$
$(y - 12)(y - 4) = 0$, so $e^x = 12$ or $e^x = 4$
$x = \ln 4$ or $x = \ln 12$
Answer: x = ln 4, x = ln 12