Day 295 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{8} 3^r$.

This is a geometric series with first term 3, common ratio 3 and 8 terms

$S = \frac{3(3^8 - 1)}{2} = 9840$

Answer: 9840

2 Binomial probabilities (All boards)

X ~ B(30, 0.5). Find, to 3 significant figures,
(a) P(X < 15)
(b) P(X = 14)

(a) $P(X < 15) = P(X \leq 14)$
(a) = 0.428
(b) $P(X = 14) = {}^{30}C_{14} \times 0.5^{14} \times 0.5^{16} = 0.135$

Answer: (a) 0.428 (b) 0.135

3 Laws of logarithms: solving equations (All boards)

Solve $\log_5(x + 27) - \log_5(x - 1) = 1$.

$\log_5(\frac{x + 27}{x - 1}) = 1$, so $\frac{x + 27}{x - 1} = 5^1 = 5$
$x + 27 = 5(x - 1) = 5x - 5$
$4x = 32$, so $x = 8$

Answer: x = 8

4 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = t^3 + t^2$ metres.
Find (a) the average velocity over the first 3 seconds, (b) the velocity at t = 3.

(a) s(3) = 36, so average velocity = 36 ÷ 3
(b) $v = ds/dt = 3t^2 + 2t$, v(3) = 33

Answer: (a) 12 m s^{-1} (b) 33 m s^{-1}

5 Conditional probability formula (All boards)

P(B) = 0.4, P(A | B) = 0.25 and P(A) = 0.4.
(a) Find $P(A \cap B)$.
(b) Find P(B | A).
(c) Find $P(A \cup B)$.

(a) $P(A \cap B) = P(A | B) \times P(B) = 0.25 \times 0.4 = 0.1$
(b) $P(B | A) = \frac{0.1}{0.4} = 0.25$
(c) $P(A \cup B) = 0.4 + 0.4 - 0.1 = 0.7$

Answer: (a) 0.1 (b) 0.25 (c) 0.7

6 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 6 and common ratio 2.
Find the smallest value of n for which the sum of the first n terms exceeds 10000.

$S_n = 6(2^n - 1) > 10000$
$2^n > 1667.67$
$n > \frac{\lg 1667.67}{\lg 2} = 10.704$
So the smallest integer n is 11

Answer: n = 11

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7 Jumping to a different journey: two-stage motion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 1.5 m s^{-2} for 6 s, then travels at constant speed for 15 s, and finally decelerates uniformly at 0.5 m s^{-2} to rest.
Find the total distance travelled and the total time of the journey.

Max speed = 1.5 × 6 = 9
Stage 1: ½ × 6 × 9 = 27; stage 2: 9 × 15 = 135
Stage 3: time = 9 ÷ 0.5 = 18, distance = ½ × 9 × 18 = 81
Answer: 243 m; 39 s

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 5x + 12y = k is a tangent to the circle
$(x + 1)^2 + (y - 2)^2 = 169$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (5, 12).
A unit step in this direction is (5, 12)/13, so the points of contact are
(−1, 2) ± 13 × (5, 12)/13 = (−1, 2) ± (5, 12)
T = (4, 14) or (−6, −10), and k = 5x + 12y gives:
k = 188 or k = −150
Answer: k = 188 or k = −150

9 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that 12 cos θ + 5 sin θ ≡ 13 cos(θ − 22.6°),
solve 12 cos θ + 5 sin θ = 5 for 0 ≤ θ ≤ 360°. (1 d.p.)

13 cos(θ − 22.6°) = 5, so $\cos(\theta - 22.6°) = \frac{5}{13}$
θ − 22.6° = ±67.4°
θ = 22.6° + 67.4° = 90.0°
θ = 22.6° − 67.4° = −44.8°, so θ = 315.2°
Answer: θ = 90.0°, 315.2°

10 Simplify using double angle formulae (All boards)

Simplify $\frac{\sin 4x}{1 + \cos 4x}$,
giving your answer as a single trigonometric term.

sin 4x = 2 sin 2x cos 2x and 1 + cos 4x = 2 $\cos^2 2x$
so the expression is $\frac{2 \sin 2x \cos 2x}{2 \cos^2 2x} = \tan 2x$
Answer: tan 2x