Day 294 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Expand and simplify $(2 + \sqrt{2})(-2 - 3\sqrt{2})$, giving your answer in the form $a + b\sqrt{2}$.

$(2 + \sqrt{2})(-2 - 3\sqrt{2}) = -4 - 6\sqrt{2} - 2\sqrt{2} - 3\sqrt{2}\times\sqrt{2}$

$\sqrt{2}\times\sqrt{2} = 2$, so the expression $= -10 - 8\sqrt{2}$

Answer: $-10 - 8\sqrt{2}$

2 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A person of mass 60 kg stands on the floor of a lift. The lift accelerates upwards at 0.6 m s^{-2}.

Take g = 9.8 m s^{-2}.

Find the normal reaction between the person and the floor.

Resolve vertically (up positive): $R - 60 \times 9.8 = 60 \times 0.6$

$R = 60(9.8 + 0.6)$

Answer: 624 N

3 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $3x^2 - 30x + 79$ in the form $a(x + p)^2 + q$.

(b) Write down the coordinates of the minimum point of $y = 3x^2 - 30x + 79$.

Take out the factor 3: $3x^2 - 30x + 79 = 3(x^2 - 10x) + 79$

$= 3[(x - 5)^2 - 25] + 79$

$= 3(x - 5)^2 + 4$

$(x - 5)^2 \geq 0$ with equality at $x = 5$, so the minimum is 4 at $x = 5$.

Answer: (a) $3(x - 5)^2 + 4$ (b) (5, 4)

4 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 5\cos 3x\,dx$,

(b) the exact value of $\int_0^{\frac{\pi}{6}} 5\cos 3x\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{5\sin 3x}{3} + c$

(b) $\left[\frac{5\sin 3x}{3}\right]_0^{\frac{\pi}{6}}$

$= \frac{5}{3}$

Answer: (a) $\frac{5\sin 3x}{3} + c$ (b) $\frac{5}{3}$

5 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=4}^{24}(3r - 3)$.

There are $24 - 4 + 1 = 21$ terms

First term = 9, last term = 69

$S = \frac{21}{2}(9 + 69) = 819$

Answer: 819

6 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The shaded region is the segment cut off by the chord AB of a circle with centre O and radius 5 cm. Angle AOB = 1.9 radians.

Find the area of the segment, to 3 s.f.

Sector area = $\frac{1}{2} \times 5^2 \times 1.9 = 23.75$

Triangle area = $\frac{1}{2} \times 5^2 \times \sin 1.9 = 11.83$

Segment = sector − triangle = 11.92 cm^2

Answer: 11.9 cm^2

7 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ax^n$. A plot of $\log_{10} y$ against $\log_{10} x$ is a straight line with intercept 0.477 and gradient 2.5.

Find a (2 s.f.) and n, then estimate y when x = 9 (3 s.f.).

$\log_{10} y = \log_{10} a + n \log_{10} x$

n = gradient = 2.5; $\log_{10} a = 0.477$, so $a = 10^{0.477} = 3.0$

$y = 2.999 \times 9^{2.5} = 729$

Answer: a = 3.0, n = 2.5, y = 729

8 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The depth of water at a pier is modelled by $D = 9 + 2\sin(\frac{\pi t}{12})$, where t is in hours after noon.

Find (a) the greatest and least values, (b) the values of t with $0 \le t \le 24$ when it equals 10 m.

(a) sin ranges from −1 to 1: greatest 9 + 2 = 11, least 9 − 2 = 7

(b) $9 + 2\sin(\frac{\pi t}{12}) = 10$, so $\sin(\frac{\pi t}{12}) = \frac{1}{2}$

$\frac{\pi t}{12} = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, so t = 2 or 10

Answer: (a) 11 m, 7 m (b) t = 2 and 10

9 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(100, 0.2). Use a normal approximation to find $P(16 \le X \le 24)$, to 3 significant figures.

Y ~ N(np, np(1 − p)) = N(20.00, 16.00), so $\sigma = \sqrt{16.00} = 4.000$

Use a continuity correction.

$P(16 \le X \le 24) \approx P(15.5 < Y < 24.5) = 0.739$

Answer: 0.739

10 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 3 m and mass 5 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.

A particle of mass 2 kg is attached 2 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(2) = 5\,g(1) + 2\,g(1.5)$

$T_2 = 39.2$

$T_1 + T_2 = 7\,g$ so $T_1 = 68.6 - 39.2$

Answer: Near A: 29.4 N; near B: 39.2 N