Day 293 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Basic trig equations (degrees) (All boards)

Solve $\cos 2x = -0.35$ for $0 \leq x \leq 360°$, giving answers to 1 d.p.

Let $\theta = 2x$, so $\cos\theta = -0.35$ with θ in range 0° to 720°
$\theta = 110.49°, 249.51°, 470.49°, 609.51°$
$x = 55.2°, 124.8°, 235.2°, 304.8°$
Answer: x = 55.2°, 124.8°, 235.2°, 304.8°

2 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of

$(x^2 + \frac{3}{x})^6$.

The general term is $C(6, k)(x^2)^{6-k}(3/x)^k$, with power of x equal to $12 - 3k$
Independent of x: $12 - 3k = 0$, so $k = 4$
Term = $15 \times 3^4 = 1215$
Answer: 1215

3 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 7 \\ 11 \\ -5 \end{pmatrix}$.

The point P lies on AB with AP : PB = 3 : 2.
Find the position vector of P.

$$\text{AB} = \begin{pmatrix} 10 \\ 10 \\ -10 \end{pmatrix}$$

$$\text{AP} = \frac{3}{5}\text{AB} = \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix}$$

$$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} 3 \\ 7 \\ -1 \end{pmatrix}$$

Answer: $\begin{pmatrix} \mathbf{3} \\ \mathbf{7} \\ \mathbf{-1} \end{pmatrix}$

4 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 12 ml. Set mean 200 ml; 16 bottles have mean 203.2 ml.
Test at the 5% level whether the true mean is greater than 200.

$H_0: \mu = 200$, $H_1: \mu > 200$. Under H_0, $\overline{X} \sim N(200, \frac{12^2}{16})$

$$z = \frac{203.2 - 200}{12/\sqrt{16}} = 1.067$$

$P(Z > 1.067) = 0.1431 > 0.05$
Do not reject H_0: there is insufficient evidence that the mean has increased
Answer: do not reject H_0: insufficient evidence of an increase in the mean

5 Equations using $\sin^2x + \cos^2x = 1$ (All boards)

Solve $3\cos^2x - 2\sin x - 3 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2x = 1 - \sin^2x$: $-3\sin^2x - 2\sin x = 0$, i.e. $3\sin^2x + 2\sin x = 0$

$(\sin x)(3\sin x + 2) = 0$, so $\sin x = 0$ or $-\frac{2}{3}$

$\sin x = 0$: $x = 0.0°, 180.0°, 360.0°$

$\sin x = -\frac{2}{3}$: $x = 221.8°, 318.2°$

Answer: $x = 0.0°, 180.0°, 221.8°, 318.2°, 360.0°$

6 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A new car costs £20000. Its value falls by 15% each year.
(a) Find its value after 3 years (nearest £).
(b) In which year does it first fall below £6000?

(a) Value $= 20000 \times 0.85^3 =$ £12,283

(b) $20000 \times 0.85^n < 6000$, so $n \lg 0.85 < \lg(6000 \div 20000)$

$n > \lg(6000 \div 20000) \div \lg 0.85 = 7.408$ (dividing by a negative number)

So $n = 8$

Answer: (a) £12,283 (b) 8 years

7 Two independent binomials (All boards)

$X \sim B(12, 0.2)$ and $Y \sim B(9, 0.1)$ are independent.
Find $P(X = 2 \text{ and } Y \le 1)$, to 3 significant figures.

$P(X = 2) = 0.2835$

$P(Y \le 1) = 0.7748$

Independent, so multiply: $0.2835 \times 0.7748 = 0.220$

Answer: 0.220

8 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 6 kg lies on a rough horizontal table (coefficient of friction 0.5). It is joined by a light string over a smooth pulley to a particle of mass 6 kg hanging freely.
Take $g = 9.8$ m s^{-2}.
The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.5 \times 6g = 6a$

Hanging: $6g - T = 6a$

Add: $6g - 3g = 12a$

$a = 2.45$

$T = 6(9.8 - 2.45)$

Answer: $a = 2.45$ m s^{-2}; $T = 44.1$ N

9 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(2, −9) and B(13, −2), and its centre lies on the x-axis. Find the centre and the equation of the circle.

Let the centre be (h, 0). Then $CA^2 = CB^2$:

$(h - 2)^2 + 81 = (h - 13)^2 + 4$

Expanding and cancelling h^2: $-4h + 85 = -26h + 173$

$h = 4$

$r^2 = CA^2 = 85$

Answer: centre (4, 0), $(x - 4)^2 + y^2 = 85$

Day 293 **A Level Maths: B Tier 10-a-day: Model Solutions**

10 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected horizontally at 205.8 m s^{-1} from a point high above the ground.
Take g = 9.8 m s^{-2}.
Find the speed and the direction of motion 20 s later, to 1 d.p. below the horizontal.

Vertical speed = 9.8 × 20 = 196
Speed = $\sqrt{(205.8^2 + 196^2)}$ = 284.2
tan α = 196 ÷ 205.8, α = 43.6°
Answer: 284.2 m s^{-1} at 43.6° below horizontal