Day 292 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}0\\-5\\-1\end{pmatrix}$ and B has position vector $\begin{pmatrix}-5\\5\\-6\end{pmatrix}$.

The point P lies on AB with AP : PB = 3 : 2.

Find the position vector of P.

$\text{AB} = \begin{pmatrix}-5\\10\\-5\end{pmatrix}$

$\text{AP} = \frac{3}{5}\text{AB} = \begin{pmatrix}-3\\6\\-3\end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix}-3\\1\\-4\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{-3}\\\mathbf{1}\\\mathbf{-4}\end{pmatrix}$

2 Conditional probability from a count (All boards)

60 people: 35 own a cat, 12 own a dog, 17 do neither.

(a) How many do both?

(b) Given a person who owns a cat, find P(this person owns a dog).

(a) At least one: $60 - 17 = 43$. Both $= 35 + 12 - 43 = 4$

(b) Of the 35 who own a cat, 4 also do the other: $\text{P} = \frac{4}{35} = \frac{4}{35}$

Answer: (a) 4 (b) $\mathbf{\frac{4}{35}}$

3 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Expand and simplify $(1 - 3\sqrt{7})(1 - 2\sqrt{7})$, giving your answer in the form $a + b\sqrt{7}$.

$(1 - 3\sqrt{7})(1 - 2\sqrt{7}) = 1 - 2\sqrt{7} - 3\sqrt{7} + 6\sqrt{7}\times\sqrt{7}$

$\sqrt{7}\times\sqrt{7} = 7$, so the expression $= 43 - 5\sqrt{7}$

Answer: $\mathbf{43 - 5\sqrt{7}}$

4 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(-24\mathbf{i} + 32\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north.

Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed $= \sqrt{(-24^2 + 32^2)} = 40$

$\tan\theta = 32 \div 24$

Answer: 40 m s^{-1}; 53.1°

5 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations

$2x + y = -10$

$x^2 + y^2 = 425$

$y = -2x - 10$; substitute: $x^2 + (-2x - 10)^2 = 425$

$5x^2 + 40x - 325 = 0$, so $x^2 + 8x - 65 = 0$

$(x - 5)(x + 13) = 0$, so $x = 5$ or $x = -13$

$y = -10 - 2x$ gives $y = -20$ and $y = 16$

Answer: x = −13, y = 16 and x = 5, y = −20

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6 Normal distribution probabilities (All boards)

The mass X g of a flour bag is normal with mean 60, s.d. 6.
Find, to 3 s.f., the probability a value is
(a) more than 54,
(b) between 54 and 57.

$X \sim N(60, 6^2)$
(a) $P(X > 54) = 0.841$
(b) $P(54 < X < 57) = 0.3085 - 0.1587 = 0.150$
Answer: (a) 0.841 (b) 0.150

7 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} -3 \\ -3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-7 + 2\lambda = -3 + 2\mu$
j: $2 + (-1)\lambda = -3 + 2\mu$
Solving i and j: $\lambda = 3$, $\mu = 1$
k check: $0 + (-1)(3) = -3$ and $-1 + (-2)(1) = -3$, equal
Point: $\begin{pmatrix} -1 \\ -1 \\ -3 \end{pmatrix}$
Answer: $\begin{pmatrix} \mathbf{-1} \\ \mathbf{-1} \\ \mathbf{-3} \end{pmatrix}$

8 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -7x - 6$ meets the circle $(x - 3)^2 + (y + 2)^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -7x - 6$: $(x - 3)^2 + (-7x - 4)^2 = 25$
$x^2 + x = 0$
$x(x + 1) = 0$, so $x = -1$ or $x = 0$
$y = -7x - 6$: A = (−1, 1), B = (0, −6)
$AB^2 = 1^2 + (-7)^2 = 50$
Answer: A(−1, 1), B(0, −6), AB = $5\sqrt{2}$

9 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $4x^2y + 3xy + 3y^3 = 48$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at (3, 1).

Differentiate with respect to x: $4x^2\frac{dy}{dx} + 8xy + 3x\frac{dy}{dx} + 9y^2\frac{dy}{dx} + 3y = 0$

$$\frac{dy}{dx} = \frac{y(-8x - 3)}{4x^2 + 3x + 9y^2}$$

At (3, 1): gradient $= -\frac{1}{2}$

Answer: $\frac{\mathbf{dy}}{\mathbf{dx}} = \frac{\mathbf{y(-8x - 3)}}{\mathbf{4x^2 + 3x + 9y^2}}$**; gradient** $\mathbf{-\frac{1}{2}}$

10 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that its position vector relative to O at time t s is r = $(3t^2)\mathbf{i} + (t^3 + 5t)\mathbf{j}$ metres.
Find the velocity when t = 1, and the speed at that time.

v = dr/dt = $(6t)\mathbf{i} + (3t^2 + 5)\mathbf{j}$
t = 1: v = $(6)\mathbf{i} + (8)\mathbf{j}$
Speed = $\sqrt{(6^2 + 8^2)}$
Answer: v = $(6)\mathbf{i} + (8)\mathbf{j}$ m s^{-1}; speed 10 m s^{-1}