Day 291 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{10}(6r + 5)$.

This is an arithmetic series with first term 11 and common difference 6
Last term = 65
$S = \frac{10}{2}(11 + 65) = 380$

Answer: 380

2 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A boat moves with constant velocity $(-24\mathbf{i} - 32\mathbf{j})$ m s^{-1}, where **i** and **j** are unit vectors east and north.
Find its speed and the acute angle its direction makes with **i**, to 1 d.p.

Speed = $\sqrt{(-24^2 + -32^2)} = 40$
$\tan\theta = 32 \div 24$
Answer: 40 m s^{-1}; 53.1°

3 Poisson: changing the interval (CIE)

Faults occur at random in a cable at an average rate of 0.5 per hour of operation.
Find the probability that in 45 minutes there are at least 1 (3 s.f.).

Rate in 45 minutes: $\lambda = 0.5 \times \frac{45}{60} = 0.375$
X ~ Po(0.375)
$P(X \geq 1) = 1 - P(X \leq 0) = 0.313$
Answer: 0.313

4 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 4t^2 - 4t + 6$ m s^{-1}.
Find the distance moved between t = 1 and t = 4. (The particle does not change direction.)

$s = \int_1^4 (4t^2 - 4t + 6)\,dt$
$= [\frac{4}{3}t^3 - 2t^2 + 6t]$ from t = 1 to t = 4
= 232/3 − 16/3
Answer: 72 m

5 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = 2xy$, given that y = 2 when x = 0.

Give y in terms of x.

$\frac{1}{y}\,dy = 2x\,dx$, so $\ln y = \frac{2x^2}{2} + c$
x = 0, y = 2: c = ln 2
$y = 2\,e^{1x^2}$
Answer: $y = 2e^{x^2}$

Day 291 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{-4x + 3}{x + 5}$ for $x \in \mathbb{R}$, $x \neq -5$.

(a) Find $f^{-1}(x)$.

(b) State the domain of f^{-1}.

Let $y = \frac{-4x + 3}{x + 5}$. Then $y(x + 5) = -4x + 3$

$xy + 5y = -4x + 3$, so $x(y + 4) = -5y + 3$

$f^{-1}(x) = \frac{-5x + 3}{x + 4}$

f never takes the value -4, so the domain of f^{-1} is $x \neq -4$.

Answer: (a) $f^{-1}(x) = \frac{-5x + 3}{x + 4}$ (b) $x \neq -4$

7 Probability without replacement (All boards)

A bag has 9 white and 6 black sweets; two are taken without replacement.

(a) Find P(same colour).

(b) Given they are the same colour, find P(both white).

(a) P(white white) $= \frac{9}{15} \times \frac{8}{14} = \frac{12}{35}$

P(black black) $= \frac{6}{15} \times \frac{5}{14} = \frac{1}{7}$

P(same) $= \frac{12}{35} + \frac{1}{7} = \frac{17}{35}$

(b) P $= \frac{12}{35} \div \frac{17}{35} = \frac{12}{17}$

Answer: (a) $\frac{17}{35}$ (b) $\frac{12}{17}$

8 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x + 5)(x + 2)(x + 1) > 0$.

Critical values: $x = -5$, $x = -2$, $x = -1$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.

Reading from the sketch: $-5 < x < -2$ or $x > -1$

Answer: $-5 < x < -2$ or $x > -1$

9 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $24 \cos\theta + 7 \sin\theta \equiv 25 \cos(\theta - 16.3°)$,
solve $24 \cos\theta + 7 \sin\theta = 1$ for $0 \leq \theta \leq 360°$. (1 d.p.)

$25 \cos(\theta - 16.3°) = 1$, so $\cos(\theta - 16.3°) = \frac{1}{25}$

$\theta - 16.3° = \pm 87.7°$

$\theta = 16.3° + 87.7° = 104.0°$

$\theta = 16.3° - 87.7° = -71.4°$, so $\theta = 288.6°$

Answer: θ = 104.0°, 288.6°

10 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-1}{(x+2)(x+3)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{-1}{(x+2)(x+3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-1}{(x+2)(x+3)} \equiv \frac{A}{x+2} + \frac{B}{x+3}$, so $-1 \equiv A(x+3) + B(x+2)$
$x = -2$: $A = -1$; $x = -3$: $B = 1$

(b) $\int_0^1 \left(\frac{-1}{x+2} + \frac{1}{x+3}\right) dx = [-\ln(x+2) + \ln(x+3)]_0^1$
$= -\ln\frac{3}{2} + \ln\frac{4}{3} = \ln\left(\frac{8}{9}\right)$

Answer: (a) $A = -1$, $B = 1$ (b) $\ln\left(\frac{8}{9}\right)$