Day 290 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-3x + 6 \leq -2x + 10$.

$-x \leq 4$

Divide by -1 (reversing the inequality): $x \geq -4$

Answer: $x \geq -4$

2 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (4e^{3x} + \frac{4}{x})\,dx$.

$\int (4e^{3x} + \frac{4}{x})\,dx = \frac{4e^{3x}}{3} + 4\ln x$

$= [\frac{4e^{3x}}{3} + 4\ln x]_1^2$

$= \ln 16 + \frac{4e^6}{3} - \frac{4e^3}{3}$

Answer: $\ln 16 + \frac{4e^6}{3} - \frac{4e^3}{3}$

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-2, 6)$ and $(8, 8)$.

(a) Find the midpoint of AB.

(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= (\frac{-2+8}{2}, \frac{6+8}{2}) = (3, 7)$

(b) Gradient $= \frac{2}{10} = \frac{1}{5}$

$y - 6 = \frac{1}{5}(x + 2)$

Multiply out and collect terms: $x - 5y + 32 = 0$

Answer: (a) (3, 7) (b) $x - 5y + 32 = 0$

4 Velocity from variable acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its acceleration is $t + 4$ m s^{-2}.

When $t = 0$ the velocity is 1 m s^{-1}. Find the velocity when $t = 2$.

$v = \int (t + 4)\,dt = \frac{1}{2}t^2 + 4t + c$

$t = 0$, $v = 1$ so $c = 1$

$t = 2$: $v = 11$

Answer: 11 m s^{-1}

5 Rates of change (connected rates) (All boards)

The area of a square increases at a constant rate of 30 cm^{2}s^{-1}.

Find the rate of increase of the side length when the side is 4 cm.

$A = x^2$, so $\frac{dA}{dt} = 2x\frac{dx}{dt}$

$30 = 2 \times 4 \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{15}{4}$

Answer: $\frac{15}{4}$ cm/s

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6 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 1 kg and 9 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest. Take g = 9.8 m s^{-2}.

Find the acceleration and the tension in the string.

Heavier: 9 g − T = 9 a
Lighter: T − 1 g = 1 a
Add: 8 g = 10 a, so a = 7.84
T = 1(9.8 + 7.84)

Answer: a = 7.84 m s^{-2}; T = 17.64 N

7 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation y = x − 6. The point P is (1, −1).

(a) Find the line through P perpendicular to l, as ax + by + c = 0 with integer a, b, c.

(b) Find the foot F and the exact distance PF.

(a) Gradient of l = 1, so the perpendicular has gradient −1
y + 1 = −(x − 1), giving x + y = 0
(b) Solve y = x − 6 with x + y = 0: F = (3, −3)
$PF^2 = (-2)^2 + 2^2 = 8$

Answer: (a) x + y = 0 (b) F(3, −3), PF = $2\sqrt{2}$

8 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{36}$ for $0 \le x \le 6$, and 0 otherwise.

(a) Find P(X > 1).

(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{36}$

(a) $P(X > 1) = 1 - F(1) = \frac{35}{36}$

(b) F(m) = 0.5: $\frac{m^2}{36} = \frac{1}{2}$

m = 4.24

Answer: (a) $\frac{35}{36}$ (b) 4.24

9 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^{\ln 3} \frac{e^x}{e^x + 4}\,dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{e^x}{e^x + 4}\,dx = \ln(e^x + 4)$

$= [\ln(e^x + 4)]_0^{\ln 3}$

$= \ln(\frac{7}{5})$

Answer: $\ln(\frac{7}{5})$

10 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The time, X minutes, to complete a task is normally distributed with mean 50 and s.d. 12.
A random sample of 4 is taken. Find the probability that the mean time is more than 62 (3 s.f.).

$\overline{X}$ ~ N(50, $\frac{12^2}{4}$), standard deviation 6.000

$z = \frac{62 - 50}{6.000} = 2.000$

P = 0.0228

Answer: 0.0228