Day 29 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 200 kg is raised and lowered by a vertical cable. The lift accelerates upwards at 1.5 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolve vertically (up positive): $T - 200 \times 9.8 = 200 \times 1.5$
$T = 200(9.8 + 1.5)$
Answer: 2260 N

2 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 53 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 53$, $\Sigma fx = 135$, $\Sigma fx^2 = 523$

Mean = $\frac{135}{53} = 2.55$

Variance = $\frac{523}{53} - (2.5472)^2 = 3.380$

Standard deviation = 1.84
Answer: mean 2.55, standard deviation 1.84

3 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $(\frac{16}{81})^{\frac{1}{4}}$.

$\frac{16}{81} = (\frac{2}{3})^4$, so the expression = $(\frac{2}{3})^1$
$= \frac{2}{3}$

Answer: $\frac{2}{3}$

4 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = \frac{1}{2}$ and θ is acute,
find the exact value of cosec θ.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = \frac{1}{2}$, so $\sec^2\theta = 1 + \frac{1}{4} = \frac{5}{4}$
$\cot\theta = 2$, so $\operatorname{cosec}^2\theta = 5$ and $\operatorname{cosec}\theta = \sqrt{5}$
Answer: $\sqrt{5}$

5 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 + 6x + 6y + 5 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:
$(x + 3)^2 - 9 + (y + 3)^2 - 9 + 5 = 0$
$(x + 3)^2 + (y + 3)^2 = 13$
Centre $(-3, -3)$, radius $\sqrt{13}$
Answer: centre $(-3, -3)$, radius $\sqrt{13}$

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6 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = x^2 \ln x$, and determine its nature.

$\frac{dy}{dx} = x(2 \ln x + 1) = 0$

$x = \frac{1}{\sqrt{e}}$

$\frac{d^2y}{dx^2} = 2 \ln x + 3$

At $x = \frac{1}{\sqrt{e}}$: $\frac{d^2y}{dx^2} = 2 > 0$, so a minimum

Answer: $(\frac{1}{\sqrt{e}}, -\frac{1}{2e})$ minimum

7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of $(1 + 4x)(1 + x)^5$.

$(1 + x)^5$ has x^1 coefficient 5 and x^2 coefficient 10

x^2 term of the product = $1 \times 10 + 4 \times 5$

= 30

Answer: 30

8 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 2000 kg moves along a straight horizontal road. The engine produces a driving force of 2400 N and the resistance to motion is 800 N.
Find the acceleration of the car.

Resultant force = 2400 − 800 = 1600 N

F = ma: 1600 = 2000 a

Answer: 0.8 m s^{-2}

9 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.4. It is tossed 40 times and lands heads 11 times.
Test, at the 5% significance level, whether the probability of heads is different from 0.4.

H_0: p = 0.4, H_1: $p \neq 0.4$. Under H_0, X ~ B(40, 0.4)

$P(X \leq 11) = 0.0709$

Two-tailed: compare with 0.025. 0.0709 > 0.025, so do not reject H_0

There is insufficient evidence that the probability of heads is different from 0.4

Answer: do not reject H_0: insufficient evidence that the probability of heads is different from 0.4

10 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = e^x + 3$ to find the exact value of

$\int_0^{\ln 2} (e^x + 3)^3 e^x \, dx$.

$\frac{du}{dx} = e^x$, so $e^x \, dx = du$

Limits: x = 0 gives u = 4, x = ln 2 gives u = 5

$\int_4^5 u^3 \, du = [\frac{u^4}{4}]_4^5$

$= \frac{369}{4}$

Answer: $\frac{369}{4}$