Day 289 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} 12 \\ 1 \\ 12 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{12^2 + 1^2 + 12^2} = \sqrt{289} = 17$

Unit vector $= \frac{1}{17}\mathbf{a}$

Answer: $|\mathbf{a}| = 17$, unit vector $\frac{1}{17}\begin{pmatrix} 12 \\ 1 \\ 12 \end{pmatrix}$

2 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of $y = f(x)$ onto the graph of:

(a) $y = f(\frac{x}{4})$

(b) $y = 3f(x)$

(c) $y = f(-x)$

(a) $y = f(\frac{x}{4})$ is a stretch, scale factor 4, parallel to the x-axis.

(b) $y = 3f(x)$ is a stretch, scale factor 3, parallel to the y-axis.

(c) $y = f(-x)$ is a reflection in the y-axis.

Answer: (a) stretch, scale factor 4, parallel to the x-axis (b) stretch, scale factor 3, parallel to the y-axis (c) reflection in the y

3 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 4t^3 + 2t^2 + 8t$ metres.
Find its velocity and acceleration when $t = 3$.

$v = ds/dt = 12t^2 + 4t + 8$

$a = dv/dt = 24t + 4$

$t = 3$: $v = 128$, $a = 76$

Answer: $v = 128$ m s^{-1}, $a = 76$ m s^{-2}

4 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^3 + 3n + 4$ is even for all integers n.

$n^3 + 3n = n(n^2 + 3)$

If n is even, $n(n^2 + 3)$ is even.

If n is odd, n^2 is odd and $n^2 + 3$ is even, so $n(n^2 + 3)$ is even.

So $n^3 + 3n$ is always even, and adding 4 gives an even number.

Answer: expression is even

5 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin\theta = \frac{24}{25}$ and $0 < \theta < 90°$,

find the exact value of $\sin 2\theta$.

$\cos\theta = \pm\sqrt{1 - (\frac{24}{25})^2} = \frac{7}{25}$ (θ is acute)

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{24}{25} \times \frac{7}{25} = \frac{336}{625}$

Answer: $\frac{336}{625}$

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6 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 55 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 5 to t = 45.
The floors are 950 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (55 + 40) × V = 950
95 V = 1900 so V = 20
Deceleration = 20 ÷ 10

Answer: V = 20; deceleration 2 m s^{-2}

7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-5x - 13}{(x + 2)(x + 3)}$ in partial fractions.

(b) Hence evaluate $\int_0^3 \frac{-5x - 13}{(x + 2)(x + 3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-5x - 13}{(x + 2)(x + 3)} \equiv \frac{A}{x + 2} + \frac{B}{x + 3}$, so $-5x - 13 \equiv A(x + 3) + B(x + 2)$
x = −2: A = −3; x = −3: B = −2

(b) $\int_0^3 \left(\frac{-3}{x + 2} + \frac{-2}{x + 3}\right) dx = [-3\ln(x + 2) - 2\ln(x + 3)]_0^3$

$= -3\ln\frac{5}{2} - 2\ln\frac{6}{3} = \ln\left(\frac{2}{125}\right)$

Answer: (a) A = −3, B = −2 (b) $\ln\left(\frac{2}{125}\right)$

8 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{64}$ for $0 \le x \le 8$, and 0 otherwise.

(a) Find P(X > 7).
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{64}$

(a) $P(X > 7) = 1 - F(7) = \frac{15}{64}$

(b) $F(m) = 0.5$: $\frac{m^2}{64} = \frac{1}{2}$

m = 5.66

Answer: (a) $\frac{15}{64}$ (b) 5.66

9 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x + 2)(x + 1)(x + 5) \ge 0$.

Critical values: x = −5, x = −2, x = −1
The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.
Reading from the sketch: $-5 \le x \le -2$ or $x \ge -1$

Answer: $-5 \le x \le -2$ or $x \ge -1$

10 Poisson: changing the interval (CIE)

Meteors are seen at random at an average rate of 8 per hour.
Find the probability that in 15 minutes there are exactly 3 (3 s.f.).

Rate in 15 minutes: $\lambda = 8 \times \frac{15}{60} = 2$

X ~ Po(2)

$P(X = 3) = e^{-2} \times \frac{2^3}{3!} = 0.180$

Answer: 0.180