Day 288 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Transformations of a turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

y = f(x) has a maximum point at (6, 2).
Find the turning point on the curve:

(a) $y = f(\frac{x}{2})$ (b) y = f(x + 3)

(c) y = 3f(x)

(a) $y = f(\frac{x}{2})$: the point (6, 2) maps to (12, 2)

(b) y = f(x + 3): the point (6, 2) maps to (3, 2)
(c) y = 3f(x): the point (6, 2) maps to (6, 6)
Answer: (a) (12, 2) (b) (3, 2) (c) (6, 6)

2 Tree diagram (All boards)

The tree diagram shows probabilities for event E, followed by event F.
Find the probability that exactly one of E and F occurs.

P(E and not F) = 0.6 × 0.8 = 0.48
P(not E and F) = 0.4 × 0.9 = 0.36
Exactly one: 0.48 + 0.36 = 0.84
Answer: 0.84

3 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(-3\mathbf{i} - \mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(-5\mathbf{i} + 4\mathbf{j})$ m.
Find (a) its position vector when t = 2, (b) the value of t when it crosses the line through O in the direction of **i**.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (-5\mathbf{i} + 4\mathbf{j}) + 2(-3\mathbf{i} - \mathbf{j}) = (-11\mathbf{i} + 2\mathbf{j})$
(b) **j** component zero: 4 − t = 0, so t = 4
Answer: (a) $(-11\mathbf{i} + 2\mathbf{j})$ m (b) t = 4 s

4 Solving exponential equations using logarithms (All boards)

Solve $5^{3x-3} = 145$, giving your answer to 3 s.f.

Take logs: $(3x - 3) \ln 5 = \ln 145$

$3x - 3 = \frac{\ln 145}{\ln 5} = 3.0922$

x = 2.031
Answer: x = 2.03

5 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic series is 2 and the common difference is 7.
The sum of the first n terms is 1817. Find the value of n.

$S_n = \frac{n}{2}(2 \times 2 + (n - 1) \times 7) = 1817$

$7n^2 - 3n - 3634 = 0$
(n − 23)(7n + 158) = 0
n must be positive, so n = 23
Answer: n = 23

6 Simplify using double angle formulae (All boards)

Simplify $\frac{1 - \cos 4x}{\sin 4x}$,
giving your answer as a single trigonometric term.

$1 - \cos 4x = 2 \sin^2 2x$ and $\sin 4x = 2 \sin 2x \cos 2x$

so the expression is $\frac{2 \sin^2 2x}{2 \sin 2x \cos 2x} = \tan 2x$

Answer: tan 2x

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7 Binomial two-tailed test (All boards)

A factory says a proportion 0.4 of its parts are defective. In a sample of 40 parts, 12 are defective.
Test, at the 5% significance level, whether the proportion is different from 0.4.

H_0: p = 0.4, H_1: p ≠ 0.4. Under H_0, X ~ B(40, 0.4)

P(X ≤ 12) = 0.1285

Two-tailed: compare with 0.025. 0.1285 > 0.025, so do not reject H_0

There is insufficient evidence that the proportion is different from 0.4

Answer: do not reject H_0: insufficient evidence that the proportion is different from 0.4

8 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A golf ball is hit from the ground at 83.3 m s^{-1} at angle θ to the horizontal, where $\sin\theta = \frac{8}{17}$.

Take g = 9.8 m s^{-2}.

Find (a) the greatest height, (b) the speed of the ball at the greatest height.

Vertical component 39.2, horizontal component 73.5

(a) $0 = 39.2^2 - 2 \times 9.8 \times H$, $H = 1536.64 \div 19.6$

(b) At the top the vertical velocity is zero, so the speed is the horizontal component.

Answer: (a) 78.4 m (b) 73.5 m s^{-1}

9 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-9x^2 - 13x}{(x-4)(x+3)^2}$ in partial fractions.

$$\frac{-9x^2 - 13x}{(x-4)(x+3)^2} \equiv \frac{A}{(x-4)} + \frac{B}{(x+3)} + \frac{C}{(x+3)^2}$$

$-9x^2 - 13x \equiv A(x+3)^2 + B(x-4)(x+3) + C(x-4)$

x = 4 gives A = −4; x = −3 gives C = 6

Coefficients of x^2: −9 = A + B, so B = −5

Answer: $-\frac{4}{(x-4)} - \frac{5}{(x+3)} + \frac{6}{(x+3)^2}$

10 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 2 cm per second.

Find the rate at which its area is increasing when r = 10 cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 2$

When r = 10: $\frac{dA}{dt} = 40\pi$

Answer: 40π cm^2/s