Day 287 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = (2x + 6)^3$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = -1$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = 24(x + 3)^2$

When $x = -1$: gradient = 96

Answer: $\frac{dy}{dx} = 24(x + 3)^2$; gradient 96

2 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the times, t minutes, taken by 7 people to complete a puzzle gives $\Sigma t = 193$ and $\Sigma t^2 = 6583$.
Find the mean and the standard deviation (3 s.f.).

Mean = $193 \div 7 = 27.6$

Variance = $\frac{6583}{7} - (27.571)^2 = 180.2$

Standard deviation = $\sqrt{180.2} = 13.4$

Answer: mean 27.6, standard deviation 13.4

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 15 cm and angle 72°.
Find the perimeter of the sector, in terms of π.

$\theta = \frac{2\pi}{5}$ radians

Arc length = $r\theta = 6\pi$ cm

Perimeter = 2r + arc = $6\pi + 30$ cm

Answer: $6\pi + 30$ cm

4 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (1, 3), (−2, 4) and (−1, 7).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB = $\frac{-1}{3} = -\frac{1}{3}$

Gradient BC = $\frac{3}{1} = 3$

$(-\frac{1}{3}) \times 3 = -1$, so AB and BC are perpendicular.

(b) AB = $\sqrt{10}$ and BC = $\sqrt{10}$

Area = $\frac{1}{2} \times AB \times BC = 5$

Answer: (b) area = 5

5 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 3 m and mass 18 kg is hinged at A and held horizontal by a light string attached at B.

The string makes an angle θ with the rod, where $\sin\theta = \frac{4}{5}$. Take $g = 9.8$ m s^{-2}. Find the tension in the string.

Moments about A: $T \sin\theta \times 3 = mg \times 1.5$

$T \times \frac{4}{5} \times 3 = 176.4 \times 1.5$

$T = 264.6 \div 2.4$

Answer: 110.25 N

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6 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{100}$ for $0 \leq x \leq 10$, and 0 otherwise.

(a) Find $P(X > 6)$.
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{100}$

(a) $P(X > 6) = 1 - F(6) = \frac{16}{25}$

(b) $F(m) = 0.5$: $\frac{m^2}{100} = \frac{1}{2}$

$m = 7.07$

Answer: (a) $\frac{16}{25}$ (b) 7.07

7 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 8t - 2t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 8 - 4t$
Maximum speed when $a = 0$: $t = 8 \div 4 = 2$
$v = 8 \times 2 - 2 \times 2^2$

Answer: 8 m s^{-1} at t = 2 s

8 Quotient rule (All boards)

Given that $y = \frac{3x^2 + 5}{x + 5}$,

find $\frac{dy}{dx}$, and its value when $x = 0$.

Quotient rule with $u = 3x^2 + 5$, $v = x + 5$
$u' = 6x$, $v' = 1$

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{6x(x + 5) - 3x^2 - 5}{(x + 5)^2}$$

When $x = 0$: $\frac{dy}{dx} = -\frac{1}{5}$

Answer: $\frac{dy}{dx} = \frac{6x(x + 5) - 3x^2 - 5}{(x + 5)^2}$; value $-\frac{1}{5}$

9 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x - 10$ meets the circle $(x + 1)^2 + y^2 = 65$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -x - 10$: $(x + 1)^2 + (-x - 10)^2 = 65$
$x^2 + 11x + 18 = 0$
$(x + 2)(x + 9) = 0$, so $x = -9$ or $x = -2$
$y = -x - 10$: A = $(-9, -1)$, B = $(-2, -8)$
$AB^2 = 7^2 + (-7)^2 = 98$

Answer: A$(-9, -1)$, B$(-2, -8)$, AB = $7\sqrt{2}$

10 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "$n^2 + n$ is a multiple of 9 for every positive integer n."
Find a counter-example.

Try n = 1, 2, 3, ... until $n^2 + n$ is not a multiple of 9.
$n = 1$: $n^2 + n = 2$, and $2 \div 9$ is not an integer.

Answer: n = 1 (value 2)