Day 286 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of a.

$|\mathbf{a}| = \sqrt{4^2 + 3^2 + 12^2} = \sqrt{169} = 13$

Unit vector $= \frac{1}{13}\mathbf{a}$

Answer: $|\mathbf{a}| = 13$, unit vector $\frac{1}{13}\begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix}$

2 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 4)^2 + (y + 1)^2 = 169$.
The point P(16, 4) lies on C.
Find an equation of the tangent to C at P, in the form $ax + by + c = 0$ with integer a, b, c.

Centre of C = (4, −1). Gradient of radius CP $= \frac{5}{12}$

The tangent is perpendicular to the radius: gradient $= -\frac{12}{5}$

$y - 4 = -\frac{12}{5}(x - 16)$

Multiply out and collect terms: $12x + 5y - 212 = 0$

Answer: $12x + 5y - 212 = 0$

3 Binomial expected frequency (All boards)

Each of 6 people independently supports a team with probability $\frac{1}{3}$; X is the number of supporters.

This is done 1000 times. Estimate the number of times that X = 3.

$X \sim B(6, \frac{1}{3})$

$P(X = 3) = {}^6C_3 \times (\frac{1}{3})^3 \times (\frac{2}{3})^3 = 0.2195$

Expected = $1000 \times 0.2195 = 219.5$

Answer: about 219 times

4 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(4\mathbf{i} + 3\mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(4\mathbf{i} + 3\mathbf{j})$ m s^{-2}.
Find its velocity and speed when t = 4.

$\mathbf{v} = \mathbf{u} + \mathbf{at} = (4\mathbf{i} + 3\mathbf{j}) + 4(4\mathbf{i} + 3\mathbf{j})$
$= 20\mathbf{i} + 15\mathbf{j}$
Speed $= \sqrt{(20^2 + 15^2)}$

Answer: $\mathbf{v} = (20\mathbf{i} + 15\mathbf{j})$ m s^{-1}; speed 25 m s^{-1}

5 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

x-intercepts at x = −4, −1 and 4, so $y = k(x + 4)(x + 1)(x - 4)$
The curve passes through (0, −16): $k \times 4 \times 1 \times (-4) = -16$
So k = 1

Answer: $f(x) = (x + 4)(x + 1)(x - 4)$

Day 286 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Double angle equations (All boards)

Solve $\cos 2x = 1 - 3\sin x$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $1 - 2\sin^2 x = 1 - 3\sin x$
$\sin x(2\sin x - 3) = 0$
$\sin x = 0$: $x = 0.0°, 180.0°, 360.0°$
$\sin x = \frac{3}{2}$ has no solutions

Answer: $x = 0.0°, 180.0°, 360.0°$

7 Two independent binomials (All boards)

$X \sim B(8, 0.4)$ and $Y \sim B(8, 0.3)$ are independent.
Find $P(X = 3 \text{ and } Y \leq 2)$, to 3 significant figures.

$P(X = 3) = 0.2787$
$P(Y \leq 2) = 0.5518$
Independent, so multiply: $0.2787 \times 0.5518 = 0.154$
Answer: 0.154

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x - 4y = k$ is a tangent to the circle
$(x + 4)^2 + (y + 1)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(3, -4)$.
A unit step in this direction is $(3, -4)/5$, so the points of contact are
$(-4, -1) \pm 10 \times (3, -4)/5 = (-4, -1) \pm (6, -8)$
T = $(2, -9)$ or $(-10, 7)$, and $k = 3x - 4y$ gives:
$k = 42$ or $k = -58$
Answer: $k = 42$ or $k = -58$

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $1 + 1\lambda = 1 + 0\mu$
j: $4 + 2\lambda = 10 + (-2)\mu$
Solving i and j: $\lambda = 0$, $\mu = 3$
k check: $-1 + 2(0) = -1$ and $-1 + 0(3) = -1$, equal
Point: $\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{1} \\ \mathbf{4} \\ \mathbf{-1} \end{pmatrix}$

Day 286 **A Level Maths: B Tier 10-a-day: Model Solutions**

10 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = t^3 - 2t^2 + 2t$ metres.
Find its velocity and acceleration when t = 4.

$v = ds/dt = 3t^2 - 4t + 2$
$a = dv/dt = 6t - 4$
t = 4: v = 34, a = 20

Answer: $v = 34\ m\ s^{-1}$, $a = 20\ m\ s^{-2}$