Day 285 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A sports club has 260 junior members, 300 senior members and 320 veteran members.
A stratified sample of size 44 is taken. Find the number selected from each group.

Total = 260 + 300 + 320 = 880

Sampling fraction = 44/880 = $\frac{1}{20}$

junior: 260 ÷ 20 = 13
senior: 300 ÷ 20 = 15
veteran: 320 ÷ 20 = 16
Answer: junior 13, senior 15, veteran 16

2 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A ship moves with constant velocity $(4\mathbf{i} - \mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(5\mathbf{i} + 7\mathbf{j})$ m. Find (a) its position vector when t = 2, (b) the value of t when it crosses the line through O in the direction of $\mathbf{i}$.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (5\mathbf{i} + 7\mathbf{j}) + 2(4\mathbf{i} - \mathbf{j}) = (13\mathbf{i} + 5\mathbf{j})$
(b) $\mathbf{j}$ component zero: $7 - t = 0$, so $t = 7$
Answer: (a) $(13\mathbf{i} + 5\mathbf{j})$ m (b) t = 7 s

3 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $\tan^2 x + 4\tan x + 3 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(\tan x + 3)(\tan x + 1) = 0$, so $\tan x = -3$ or $\tan x = -1$
$\tan x = -3$: x = 108.4°, 288.4°
$\tan x = -1$: x = 135.0°, 315.0°
Answer: x = 108.4°, 135.0°, 288.4°, 315.0°

4 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{-2}{x - 3} - 1$.

(a) Write down the asymptotes.
(b) Find where the curve crosses the axes.

(a) The denominator is zero when x = 3, and $y \to -1$ as $x \to \pm\infty$
Asymptotes: x = 3 and y = −1
(b) x = 0: $y = \frac{2}{3} - 1 = -\frac{1}{3}$
y = 0: $\frac{-2}{x - 3} = 1$, so $x - 3 = -2$, x = 1
Answer: (a) x = 3, y = −1 (b) $(0, -\frac{1}{3})$ and (1, 0)

5 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a}\cdot\mathbf{b} = (5)(5) + (2)(-4) + (4)(2) = 25$
$|\mathbf{a}| = \sqrt{45}$, $|\mathbf{b}| = \sqrt{45}$
$\cos\theta = \frac{25}{\sqrt{45}\sqrt{45}} = 0.5556$
$\theta = 56.3°$
Answer: 56.3°

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6 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 4 kg is acted on by forces $(\mathbf{i} - \mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(3\mathbf{i} - 2\mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(\mathbf{i} - \mathbf{j}) + \mathbf{P} = 4(3\mathbf{i} - 2\mathbf{j}) = (12\mathbf{i} - 8\mathbf{j})$
$\mathbf{P} = 11\mathbf{i} - 7\mathbf{j}$
$|\mathbf{P}| = \sqrt{(11^2 + -7^2)}$
Answer: $\mathbf{P} = (11\mathbf{i} - 7\mathbf{j})$ N; $|\mathbf{P}| = 13.0$ N

7 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{u_n - 1}{u_n + 1}$, $u_1 = \frac{1}{2}$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{30} and $\sum_{r=1}^{30} u_r$.

(a) $u_2 = -\frac{1}{3}$, $u_3 = -2$, $u_4 = 3$, $u_5 = \frac{1}{2}$
$u_5 = u_1$ so the sequence repeats with period 4

(b) $30 = 4 \times 7 + 2$, so $u_{30} = u_2 = -\frac{1}{3}$

Sum of one cycle $= \frac{7}{6}$, so the sum $= 7 \times \frac{7}{6} + \frac{1}{6} = \frac{25}{3}$

Answer: period 4; $u_{30} = -\frac{1}{3}$; sum $= \frac{25}{3}$

8 Binomial: smallest n (All boards)

Given that each attempt at a task succeeds independently with probability 0.3, find the least number of attempts needed for the probability of at least one success to exceed 0.9.

P(at least one) $= 1 - 0.7^n > 0.9$
$0.7^n < 0.1$
$n > \frac{\log 0.1}{\log 0.7} = 6.456$
Answer: n = 7

9 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$2x + y = -10$
$x^2 + y^2 = 25$

$y = -2x - 10$; substitute: $x^2 + (-2x - 10)^2 = 25$
$5x^2 + 40x + 75 = 0$, so $x^2 + 8x + 15 = 0$
$(x + 5)(x + 3) = 0$, so $x = -5$ or $x = -3$
$y = -10 - 2x$ gives $y = 0$ and $y = -4$
Answer: $x = -5$, $y = 0$ and $x = -3$, $y = -4$

10 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_1^3 \ln x \, dx$.

Let $u = \ln x$ and $\frac{dv}{dx} = 1$, so $\frac{du}{dx} = \frac{1}{x}$ and $v = x$

Integral $= [uv] - \int v\frac{du}{dx} \, dx$

$= [x \ln x]_1^3 - \int_1^3 1 \, dx$
$= \ln 27 - 2$
Answer: ln 27 − 2