Day 284 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 5\sec^2 4x\,dx$,

(b) the exact value of $\int_0^{\frac{\pi}{16}} 5\sec^2 4x\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{5\tan 4x}{4} + c$

(b) $\left[\frac{5\tan 4x}{4}\right]_0^{\frac{\pi}{16}}$

$= \frac{5}{4}$

Answer: (a) $\frac{5\tan 4x}{4} + c$ (b) $\frac{5}{4}$

2 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A van of mass 2000 kg moves along a straight horizontal road. The engine produces a driving force of 4000 N and the resistance to motion is 500 N.
Find the acceleration of the van.

Resultant force = 4000 − 500 = 3500 N
F = ma: 3500 = 2000 a
Answer: 1.75 m s^{-2}

3 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 5t$, $y = \frac{7}{t}$, $t \neq 0$.

Find a Cartesian equation of the curve.

Multiply the two equations: $xy = 5t \times \frac{7}{t} = 35$

So $y = \frac{35}{x}$

Answer: $y = \frac{35}{x}$

4 Binomial one-tailed test (All boards)

A seed supplier says that 0.25 of seeds germinate. In a sample of 15 seeds, 8 germinate.
Test, at the 5% level, whether the true proportion is higher than claimed.

H_0: p = 0.25, H_1: p > 0.25. Let X ~ B(15, 0.25) under H_0
$P(X \geq 8) = 1 - P(X \leq 7) = 0.0173$
0.0173 < 0.05, so reject H_0
There is sufficient evidence that the proportion is higher than claimed
Answer: reject H_0: evidence that the proportion is higher

5 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 10 kg is pulled across smooth horizontal ice by a rope with tension 26 N, inclined at α above the horizontal, where $\tan\alpha = \frac{3}{4}$.

Take g = 9.8 m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

cos α = 4/5, sin α = 3/5
(a) Horizontal: 26 × 4/5 = 10 a
(b) Vertical: R + 26 × 3/5 = 10 × 9.8
Answer: (a) 2.08 m s^{-2} (b) 82.4 N

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6 Binomial mean and variance (All boards)

X has a binomial distribution with mean 4.8 and variance 2.88.
(a) Find n and p.
(b) Find P(X = 4) to 3 significant figures.

(a) $np = 4.8$ and $np(1 - p) = 2.88$
$1 - p = 2.88 \div 4.8 = 0.6$, so $p = 0.4$ and $n = 4.8 \div 0.4 = 12$
(b) $P(X = 4) = {}^{12}C_4 \times 0.4^4 \times 0.6^8 = 0.213$
Answer: (a) n = 12, p = 0.4 (b) 0.213

7 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\cot^2 x + 9\operatorname{cosec} x + 11 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $2\operatorname{cosec}^2 x + 9\operatorname{cosec} x + 9 = 0$
$(2\operatorname{cosec} x + 3)(\operatorname{cosec} x + 3) = 0$, so $\operatorname{cosec} x = -\frac{3}{2}$ or -3
$\operatorname{cosec} x = -\frac{3}{2}$, so $\sin x = -\frac{2}{3}$: $x = 221.8°, 318.2°$
$\operatorname{cosec} x = -3$, so $\sin x = -\frac{1}{3}$: $x = 199.5°, 340.5°$
Answer: x = 199.5°, 221.8°, 318.2°, 340.5°

8 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + px^2 + qx + 9$.
When f(x) is divided by $(x - 2)$ the remainder is 9, and when divided by $(x + 1)$ the remainder is -3. Find p and q.

$f(2) = 9$: $16 + 4p + 2q + 9 = 9$, so $4p + 2q = -16$
$f(-1) = -3$: $-2 + p - q + 9 = -3$, so $p - q = -10$
Solving simultaneously: $p = -6$, $q = 4$
Answer: p = −6, q = 4

9 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} -1 \\ -2 \\ -4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 1 \\ -1 \end{pmatrix}$.
Find the angle between a and b, to 1 d.p.

$\mathbf{a} \cdot \mathbf{b} = (-1)(-4) + (-2)(1) + (-4)(-1) = 6$
$|\mathbf{a}| = \sqrt{21}$, $|\mathbf{b}| = \sqrt{18}$
$\cos\theta = \frac{6}{\sqrt{21}\sqrt{18}} = 0.3086$
$\theta = 72.0°$
Answer: 72.0°

10 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{4x - 8} + 5$.
(a) Find the largest possible domain of f.
(b) Find the range of f.

The expression under the root must be non-negative: $4x - 8 \geq 0$
$4x \geq 8$ gives $x \geq 2$
$\sqrt{4x - 8} \geq 0$, so $f(x) \geq 5$
Answer: (a) x ≥ 2 (b) f(x) ≥ 5