Day 283 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = e^{x^2 - 2x}$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = 0$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = 2(x - 1)e^{x^2 - 2x}$

When $x = 0$: gradient $= -2$

Answer: $\frac{dy}{dx} = 2(x - 1)e^{x^2 - 2x}$; gradient -2

2 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration increases its speed from 4 m s^{-1} to 10 m s^{-1} while covering 28 m.

Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $100 = 16 + 56\,a$, so $a = 1.5$

(b) $v = u + at$: $10 = 4 + 1.5\,t$

$t = 6 \div 1.5 = 4$

Answer: (a) 1.5 m s^{-2} (b) 4 s

3 Locating roots: change of sign (All boards)

The function $f(x) = x^3 - 3x - 7$.

Show that $f(x) = 0$ has a root between $x = 2$ and $x = 2.5$.

$f(2) = -5.00$

$f(2.5) = 1.12$

f is continuous and changes sign, so there is a root in the interval

Answer: f(2) < 0 and f(2.5) > 0: sign change

4 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (−3, −4), (0, 0) and (−8, 6).

(a) Show that angle ABC = 90°.

(b) Find the area of triangle ABC.

(a) Gradient AB $= \frac{-4}{-3} = \frac{4}{3}$

Gradient BC $= \frac{6}{-8} = -\frac{3}{4}$

$\frac{4}{3} \times (-\frac{3}{4}) = -1$, so AB and BC are perpendicular.

(b) AB = 5 and BC = 10

Area $= \frac{1}{2} \times$ AB $\times$ BC $= 25$

Answer: (b) area = 25

5 Binomial critical region (All boards)

$X \sim B(20, p)$. H_0: $p = 0.2$ is to be tested against H_1: $p > 0.2$ at the 5% significance level.

Find the critical region and the actual significance level.

Under H_0, $X \sim B(20, 0.2)$

$P(X \geq 8) = 0.0321 \leq 0.05$

$P(X \geq 7) = 0.0867 > 0.05$

The critical region is $X \geq 8$

Answer: $X \geq 8$; actual significance level 3.21%

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6 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = 3t^3 + 3t^2$ metres.
Find (a) the average velocity over the first 3 seconds, (b) the velocity at t = 3.

(a) s(3) = 108, so average velocity = 108 ÷ 3

(b) $v = ds/dt = 9t^2 + 6t$, v(3) = 99

Answer: (a) 36 m s^{-1} (b) 99 m s^{-1}

7 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + x)^{-2}$ up to the x^2 term.

(b) Use x = 0.03 to estimate $\frac{1}{1.03^2}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = -2$, $a = 1$:

$= 1 - 2x + 3x^2$

(b) $1 + 1 \times 0.03 = 1.03$, so substitute x = 0.03:

$1 - 2 \times 0.03 + 3 \times 0.03^2 = 0.94270$

Answer: (a) $1 - 2x + 3x^2 + \ldots$ (b) 0.94270

8 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(200, 0.5). Use a normal approximation to find P(X ≥ 104), to 3 significant figures.

Y ~ N(np, np(1 − p)) = N(100.0, 50.00), so $\sigma = \sqrt{50.00} = 7.071$
Use a continuity correction.
P(X ≥ 104) ≈ P(Y > 103.5) = P(Z > 0.4950) = 0.310
Answer: 0.310

9 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = (5 + y)e^{-x}$,

given that y = 2 when x = 0.

$\frac{1}{5+y}\,dy = e^{-x}\,dx$, so $\ln(5 + y) = -e^{-x} + c$

x = 0: $\ln 7 = -1 + c$, so $c = 1 + \ln 7$

$5 + y = 7\,e^{1 - e^{-x}}$, so $y = 7\,e^{1 - e^{-x}} - 5$

Answer: $y = 7e^{1 - e^{-x}} - 5$

10 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + x^2 - 16x + k$.
Given that (x + 1) is a factor of f(x),
(a) find the value of k,
(b) factorise f(x) completely.

f(−1) = 0: $-1 + 1 + 16 + k = 0$

$k = -16$

$f(x) = (x + 1)(x^2 - 16)$

$= (x - 4)(x + 1)(x + 4)$

Answer: (a) $k = -16$ (b) $f(x) = (x - 4)(x + 1)(x + 4)$