**Day 282** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of cosec 30°,
giving your answer with a rational denominator.

$\text{cosec } 30° = \frac{1}{\sin 30°}$

$\sin 30° = \frac{1}{2}$, so $\text{cosec } 30° = \frac{1}{\frac{1}{2}}$

**Answer: 2**

## 2 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, texts sent in a day by 8 students:
12, 13, 16, 17, 23, 39, 39, 40.
Find $Q_1$, $Q_3$, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = \frac{29}{2}$, $Q_3 = 39$, $\text{IQR} = \frac{49}{2}$

$\text{Lower limit} = \frac{29}{2} - 1.5 \times \frac{49}{2} = -\frac{89}{4}$

$\text{Upper limit} = 39 + 1.5 \times \frac{49}{2} = \frac{303}{4}$

No value lies outside these limits.

**Answer: $Q_1 = \frac{29}{2}$, $Q_3 = 39$, IQR = $\frac{49}{2}$; no outliers**

## 3 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 8 and the coefficient of $x^2$ is 24.
Find the values of n and k.

$nk = 8$, so $k = \frac{8}{n}$

$\frac{n(n-1)}{2}k^2 = 24$, so $\frac{n(n-1)}{2} \times \frac{64}{n^2} = 24$

$\frac{(n-1) \times 64}{2n} = 24$, so $64(n-1) = 48n$

$n = 4$, and $k = 2$

**Answer: n = 4, k = 2**

## 4 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 3t^3 - 4t^2 + 4t$ metres.
Find its velocity and acceleration when t = 3.

$v = ds/dt = 9t^2 - 8t + 4$
$a = dv/dt = 18t - 8$
$t = 3$: $v = 61$, $a = 46$

**Answer: $v = 61\ \text{m s}^{-1}$, $a = 46\ \text{m s}^{-2}$**

## 5 Area under a curve (All boards)

The diagram shows the curve $y = x(4 - x)$ and the line $y = 2x$.
Find the exact area of the shaded region R.

Intersect: $x(4 - x) = 2x$, so $x = 0$ or $x = 2$

$\text{Area} = \int_0^2 (x(4 - x) - 2x)\,dx = \int_0^2 (2x - x^2)\,dx$

$= [x^2 - \frac{x^3}{3}]_0^2 = \frac{4}{3}$

**Answer: $\frac{4}{3}$**

## 6 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A rocket is projected from ground level with initial velocity components 20 m $s^{-1}$ horizontally and 39.2 m $s^{-1}$ vertically upwards.

Take g = 9.8 m $s^{-2}$.

Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 39.2^2 - 2 \times 9.8 \times H$, $H = 1536.64 \div 19.6$

(b) At the top the vertical velocity is zero, so the speed is the horizontal component.

**Answer: (a) 78.4 m (b) 20 m $s^{-1}$**

## 7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 4 g. Claimed mean 50 g; 40 packets have mean 50.9 g.

Test at the 5% level whether the true mean is greater than 50.

$H_0$: $\mu = 50$, $H_1$: $\mu > 50$. Under $H_0$, $\overline{X} \sim N(50, \frac{4^2}{40})$

$z = \frac{50.9 - 50}{4/\sqrt{40}} = 1.423$

$P(Z > 1.423) = 0.0774 > 0.05$

Do not reject $H_0$: there is insufficient evidence that the mean has increased

**Answer: do not reject $H_0$: insufficient evidence of an increase in the mean**

## 8 Exponential growth and decay models (All boards)

The mass of a radioactive sample, M grams, after t days is $M = 50\,e^{-0.2t}$.

Find (a) the half-life, (b) the rate of decrease in mass when t = 5, each to 3 s.f.

(a) $50\,e^{-0.2t} = 25$: $e^{-0.2t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.2} = 3.466$ days

(b) $\frac{dM}{dt} = -0.2 \times 50\,e^{-0.2t} = -10\,e^{-0.2t}$

At t = 5: $\frac{dM}{dt} = -10\,e^{-1} = -3.679$

so mass is decreasing at 3.68 g per day

**Answer: (a) 3.47 days (b) 3.68 g/day**

## 9 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3 \leq 4x - 4 < 12$.

Subtract $-4$ from each part: $7 \leq 4x < 16$

Divide each part by 4: $\frac{7}{4} \leq x < 4$

**Answer: $\frac{7}{4} \leq x < 4$**

## 10 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = 2x + 10$. The point P is $(-5, 5)$.

(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.

(b) Find the foot F and the exact distance PF.

(a) Gradient of l = 2, so the perpendicular has gradient $-\frac{1}{2}$

$y - 5 = -\frac{1}{2}(x + 5)$, giving $x + 2y - 5 = 0$

(b) Solve $y = 2x + 10$ with $x + 2y - 5 = 0$: F = $(-3, 4)$

$PF^2 = (-2)^2 + 1^2 = 5$

**Answer: (a) $x + 2y - 5 = 0$ (b) F$(-3, 4)$, PF = $\sqrt{5}$**