Day 281 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate

$\int_0^2 e^{-x^2}\,dx$,

giving your answer to 3 significant figures.

$h = \frac{2-0}{4} = 0.5$

x: 0, 0.5, 1, 1.5, 2
y: 1.000, 0.7788, 0.3679, 0.1054, 0.01832

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2} \times [1.000 + 0.01832 + 2 \times 1.252] = 0.8806$

Answer: 0.881

2 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 2 kg is pulled along rough horizontal ground by a horizontal force of 8.9 N. It accelerates at 2 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the coefficient of friction between the box and the ground.

R = mg = 2 × 9.8 = 19.6
Resultant: 8.9 − F = 2 × 2, so F = 4.9
F = μR: μ = 4.9 ÷ 19.6
Answer: 0.25

3 Normal distribution probabilities (All boards)

A journey takes X minutes, normal with mean 45, s.d. 8.
Find, to 3 s.f., the probability a value is
(a) less than 53,
(b) between 53 and 57.

$X \sim N(45, 8^2)$
(a) P(X < 53) = 0.841
(b) P(53 < X < 57) = 0.9332 − 0.8413 = 0.0918
Answer: (a) 0.841 (b) 0.0918

4 Work-energy: sliding to rest (CIE)

A block of mass 5 kg is projected along a rough horizontal surface with speed 7 m s^{-1} and comes to rest after sliding 10 m.
Take g = 9.8 m s^{-2}.
Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE
$\mu \times 5 \times 9.8 \times 10 = \frac{1}{2} \times 5 \times 7^2$
490 μ = 122.5
Answer: 0.25

5 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 4x − 3y = k is a tangent to the circle
$(x-2)^2 + (y+2)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (4, −3).
A unit step in this direction is (4, −3)/5, so the points of contact are
(2, −2) ± 10 × (4, −3)/5 = (2, −2) ± (8, −6)
T = (10, −8) or (−6, 4), and k = 4x − 3y gives:
k = 64 or k = −36
Answer: k = 64 or k = −36

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6 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find y in terms of x, given that $\frac{dy}{dx} = 4y^2e^x$

and y = 0.5 when x = 0.

$\frac{1}{y^2}\,dy = 4e^x\,dx$, so $-\frac{1}{y} = 4e^x + c$

x = 0, y = $\frac{1}{2}$: $-2 = 4 + c$, so $c = -6$

$\frac{1}{y} = 6 - 4e^x$, so $y = \frac{1}{6 - 4e^x}$

Answer: $y = \frac{1}{(6 - 4e^x)}$

7 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 200 and common ratio 0.75.
Find the smallest n for which the nth term is less than 0.01.

$200 \times 0.75^{n-1} < 0.01$
$(n - 1)\lg 0.75 < \lg(0.01 \div 200)$
lg 0.75 is negative, so dividing reverses the inequality:
$n - 1 > \lg(0.01 \div 200) \div \lg 0.75 = 34.425$, so $n > 35.425$
The smallest integer n is 36
Answer: n = 36

8 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The depth of water at a pier is modelled by $D = 4 + 4\sin(\frac{\pi t}{6})$, where t is in hours after noon.

Find (a) the greatest and least values, (b) the values of t with $0 \le t \le 12$ when it equals 2 m.

(a) sin ranges from -1 to 1: greatest $4 + 4 = 8$, least $4 - 4 = 0$

(b) $4 + 4\sin(\frac{\pi t}{6}) = 2$, so $\sin(\frac{\pi t}{6}) = -\frac{1}{2}$

$\frac{\pi t}{6} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, so t = 7 or 11

Answer: (a) 8 m, 0 m (b) t = 7 and 11

9 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{3x^2}{64}$ for $0 \le x \le 4$, and 0 otherwise.

(a) Find P(X > 3).
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^3}{64}$

(a) $P(X > 3) = 1 - F(3) = \frac{37}{64}$

(b) $F(m) = 0.5$: $\frac{m^3}{64} = \frac{1}{2}$

m = 3.17

Answer: (a) $\frac{37}{64}$ (b) 3.17

10 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -2x - 14$ meets the circle $(x + 2)^2 + y^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -2x - 14$: $(x + 2)^2 + (-2x - 14)^2 = 25$

$x^2 + 12x + 35 = 0$

$(x + 5)(x + 7) = 0$, so $x = -7$ or $x = -5$

$y = -2x - 14$: A = $(-7, 0)$, B = $(-5, -4)$

$AB^2 = 2^2 + (-4)^2 = 20$

Answer: A$(-7, 0)$, B$(-5, -4)$, AB = $2\sqrt{5}$