Day 280 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "If $x^2 > 64$ then $x > 8$."
Give a counter-example to show the statement is false.

$x^2 > 64$ is also true for negative x with $x < -8$.
Take $x = -9$: $x^2 = 81 > 64$, but -9 is not greater than 8.
Answer: $x = -9$

2 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 8 kg rests on a rough horizontal floor, with coefficient of friction 0.6. A horizontal force of 56 N is applied to the box.
Take $g = 9.8$ m s^{-2}.
Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 8 \times 9.8 = 78.4$, maximum friction $\mu R = 0.6 \times 78.4 = 47.04$
$56 > 47.04$ so the box moves, $F = 47.04$
$56 - 47.04 = 8a$
Answer: It moves; $a = 1.12$ m s^{-2}

3 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -8 \\ 4 \\ -8 \end{pmatrix}$.
Find $|\mathbf{a}|$ and a unit vector in the direction of a.

$|\mathbf{a}| = \sqrt{8^2 + 4^2 + 8^2} = \sqrt{144} = 12$
Unit vector $= \frac{1}{12}\mathbf{a}$
Answer: $|\mathbf{a}| = 12$, unit vector $\frac{1}{12}\begin{pmatrix} -8 \\ 4 \\ -8 \end{pmatrix}$

4 Poisson distribution (CIE)

X ~ Po(4), modelling the number of emails received per hour.
Find, to 3 significant figures,
(a) $P(X \geq 3)$
(b) $P(X \leq 3)$

(a) $P(X \geq 3) = 1 - P(X \leq 2) = 0.762$
(b) $P(X \leq 3) = P(X = 0) + ... + P(X = 3) = 0.433$
Answer: (a) 0.762 (b) 0.433

5 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The height of the tide in a harbour is modelled by $H = 7 + 4\sin(\frac{\pi t}{6})$, where t is in hours after midnight.
Find (a) the greatest and least values, (b) the values of t with $0 \leq t \leq 12$ when it equals 5 m.

(a) sin ranges from -1 to 1: greatest $7 + 4 = 11$, least $7 - 4 = 3$
(b) $7 + 4\sin(\frac{\pi t}{6}) = 5$, so $\sin(\frac{\pi t}{6}) = -\frac{1}{2}$
$\frac{\pi t}{6} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, so $t = 7$ or 11
Answer: (a) 11 m, 3 m (b) t = 7 and 11

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6 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $x^2 + 2kx + (k + 2) = 0$ has real roots.
Find the possible values of k.

$b^2 - 4ac = (2k)^2 - 4(k + 2) = 4k^2 - 4k - 8$
$= 4(k^2 - k - 2) = 4(k - 2)(k + 1)$
Need $b^2 - 4ac \geq 0$, so $k \leq -1$ or $k \geq 2$.
Answer: $k \leq -1$ or $k \geq 2$

7 Exponential growth and decay models (All boards)

A population is modelled by $N = N_0 e^{kt}$, where t is in hours. Initially N = 1000 and after 5 hours N = 2500.
Find (a) k to 3 s.f., (b) when N reaches 20000, to 3 s.f.

$N_0 = 1000$, so $2500 = 1000\, e^{5k}$, $e^{5k} = 2.5$

(a) $k = \frac{1}{5} \ln 2.5 = 0.1833$

(b) $1000\, e^{kt} = 20000$, so $t = \frac{\ln 20}{k} = 16.35$ hours

Answer: (a) k = 0.183 (b) t = 16.3 hours

8 Inverse normal (All boards)

The length X cm of a metal rod is normal with mean 500, s.d. 3.
Find k (3 s.f.): (a) P(X < k) = 0.25,
(b) P(X > k) = 0.2.

(a) $z = -0.6745$, so $k = 500 + (-0.6745) \times 3 = 498$
(b) $P(X < k) = 0.8$, $z = 0.8416$, so $k = 500 + 0.8416 \times 3 = 503$
Answer: (a) 498 (b) 503

9 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{3x^2 + 4x - 46}{(x + 5)(x - 4)}$ in the form $P + \frac{Q}{(x + 5)} + \frac{R}{(x - 4)}$.

$3x^2 + 4x - 46 = 3(x + 5)(x - 4) + (x + 14)$, so $P = 3$
$\frac{3x^2 + 4x - 46}{(x + 5)(x - 4)} \equiv 3 + \frac{Q}{(x + 5)} + \frac{R}{(x - 4)}$
$x = -5$ gives $Q = -1$; $x = 4$ gives $R = 2$
Answer: $3 + -\frac{1}{(x + 5)} + \frac{2}{(x - 4)}$

10 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 45 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 20 to t = 40.
The floors are 650 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (45 + 20) × V = 650
65 V = 1300 so V = 20
Deceleration = 20 ÷ 5
Answer: V = 20; deceleration 4 m s^{-2}