Day 28 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + px^2 + qx - 2$.
When f(x) is divided by $(x - 1)$ the remainder is 3, and when divided by $(x - 3)$ the remainder is 55. Find p and q.

$f(1) = 3$: $2 + p + q - 2 = 3$, so $p + q = 3$
$f(3) = 55$: $54 + 9p + 3q - 2 = 55$, so $9p + 3q = 3$
Solving simultaneously: $p = -1$, $q = 4$
Answer: $p = -1$, $q = 4$

2 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 50 kW. The resistance to motion is constant at 500 N.
Find the acceleration when the speed is 25 m s^{-1}.

$P = Fv$: $F = 50000 \div 25 = 2000$ N
$F - R = ma$: $2000 - 500 = 1000\,a$
Answer: 1.5 m s^{-2}

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-3, 1)$ and $(-4, -5)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{-3 + (-4)}{2}, \frac{1 + (-5)}{2}\right) = \left(-\frac{7}{2}, -2\right)$

(b) Gradient $= \frac{-6}{-1} = 6$

$y - 1 = 6(x + 3)$
Multiply out and collect terms: $6x - y + 19 = 0$

Answer: (a) $\left(-\frac{7}{2}, -2\right)$ (b) $6x - y + 19 = 0$

4 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

10 values of x are coded using $y = \frac{(x - 200)}{5}$.

$\Sigma y = 2$ and $\Sigma y^2 = 226$.
Find the mean and standard deviation of x (3 s.f.).

Mean of $y = 2 \div 10 = 0.2000$
Mean of $x = 200 + 5 \times 0.2000 = 201$
Variance of $y = \frac{226}{10} - (0.2000)^2 = 22.56$
SD of $y = 4.750$, so SD of $x = 5 \times 4.750 = 23.7$
Answer: mean 201, standard deviation 23.7

5 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 4 kg hangs horizontally from two vertical strings attached 1 m from A and 1 m from B.
A particle of mass 1 kg is attached 1.5 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(2) = 4\,g(1) + 1\,g(0.5)$
$T_2 = 22.05$
$T_1 + T_2 = 5\,g$ so $T_1 = 49 - 22.05$
Answer: Near A: 26.95 N; near B: 22.05 N

6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 1?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(−2, −1) becomes (−2, 1)
(b) The maximum of the middle part is 1. Compare k = 1 with 1:
k = 1 gives 3 intersections with y = k.
Answer: (a) (−2, 1) (b) 3

7 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = Ae^{kx}$. A graph of ln y against x is a straight line through (0, 0.5) and (5, 3).
Find A to 3 s.f. and k.

ln y = ln A + kx, so ln A = 0.5 and $A = e^{0.5} = 1.649$

k = gradient = $\frac{3 - 0.5}{5 - 0} = 0.5$

Answer: A = 1.65, k = 0.5

8 Finding the mean or standard deviation (All boards)

The mass X g of a flour bag is normal with s.d. 15.
Given P(X > 95.3) = 0.05, find the mean (3 s.f.).

P(X > 95.3) = 0.05, so P(X < 95.3) = 0.95

z = −1.645: $\frac{95.3 - \mu}{15} = -1.645$

$\mu = 95.3 + 1.645 \times 15 = 120$

Answer: μ = 120

9 Inequalities with exponents and logs (All boards)

Solve $7^x < 82$, giving your answer to 3 s.f.

Take logs: x ln 7 < ln 82

$x < \frac{\ln 82}{\ln 7} = 2.265$

Answer: x < 2.26

10 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sector OAB of a circle with centre O, radius 12 cm and angle 1.1 radians.
Find (a) the arc length AB, (b) the area of the sector.

Arc length = $r\theta = 12 \times 1.1 = 13.2$ cm

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 12^2 \times 1.1 = 79.2$ cm^2

Answer: (a) 13.2 cm (b) 79.2 cm^2