Day 279 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 91; systematic sample of size 13.
(a) Find the sampling interval.
(b) The first item is number 7. Find the 6-th item.

(a) Interval = 91 ÷ 13 = 7
(b) 6-th item = 7 + 5 × 7 = 42
Answer: interval 7; item number 42

2 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = e^x - 6x$. Use the Newton-Raphson method with
$x_0 = 0.2$ to find x_1 and x_2, to 4 d.p.

$f'(x) = e^x - 6$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 0.2 - \frac{0.02140}{-4.779} = 0.2045$$

$$x_2 = 0.2045 - \frac{0.00001227}{-4.773} = 0.2045$$

Answer: $x_1 = 0.2045$, $x_2 = 0.2045$

3 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is (5, 3) and B is (3, −3).
Find an equation of the circle in the form
$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = (4, 0)
Radius$^2 = (5 - 4)^2 + (3 - 0)^2 = 10$
Answer: $(x - 4)^2 + y^2 = 10$

4 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 6 kg is pulled along rough horizontal ground by a horizontal force of 17.7 N. It accelerates at 0.5 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the coefficient of friction between the crate and the ground.

R = mg = 6 × 9.8 = 58.8
Resultant: 17.7 − F = 6 × 0.5, so F = 14.7
F = μR: μ = 14.7 ÷ 58.8
Answer: 0.25

5 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-x + 2| \geq 8$.

$|-x + 2| \geq 8$ means $-x + 2 \leq -8$ or $-x + 2 \geq 8$
Solve each part (dividing by −1, reversing signs): $x \leq -6$ or $x \geq 10$
Answer: $x \leq -6$ or $x \geq 10$

6 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB subtends an angle of 0.8 radians at the centre O of a circle of radius 12 cm.
Find the perimeter of the minor segment, to 3 s.f.

Arc = $r\theta = 12 \times 0.8 = 9.6$

Chord AB = $2r \sin(\frac{\theta}{2}) = 9.346$

Perimeter = 9.6 + 9.346 = 18.95 cm
Answer: 18.9 cm

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7 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trunk of mass 15 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{3}{5}$.

The coefficient of friction is 0.5. Take g = 9.8 m s^{-2}. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 147 - \frac{4}{5}T$

Horizontal: $T\cos\alpha = \mu R$: $\frac{3}{5}T = 0.5(147 - \frac{4}{5}T)$

$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.5 \times 15 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)

Answer: 73.5 N

8 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix}1\\4\\0\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}1\\-2\\-1\end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a}\cdot\mathbf{b} = (1)(1) + (4)(-2) + (0)(-1) = -7$

$|\mathbf{a}| = \sqrt{17}$, $|\mathbf{b}| = \sqrt{6}$

$\cos\theta = \frac{-7}{\sqrt{17}\sqrt{6}} = -0.6931$

$\theta = 133.9°$

Answer: 133.9°

9 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.

P(X < 48) = 0.1587 and P(X > 84) = 0.0228.

Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.999$

$48 = \mu + (-0.9998)\sigma$ and $84 = \mu + 1.999\sigma$

Subtract: $36 = 2.999\,\sigma$, so $\sigma = 12.0$

$\mu = 48 - (-0.9998)(12.00) = 60.0$

Answer: μ = 60.0, σ = 12.0

10 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.

(a) Find the coordinates of the local maximum point of y = |f(x)|.

(b) How many solutions has |f(x)| = 19?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:

(0, −16) becomes (0, 16)

(b) The maximum of the middle part is 16. Compare k = 19 with 16:

k > 16 gives 2 intersections with y = k.

Answer: (a) (0, 16) (b) 2