Day 278 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train travelling at 24 m s^{-1} brakes and comes to rest with constant deceleration 1.25 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 24^2 - 2 \times 1.25 \times s$
$s = 576 \div 2.5 = 230.4$
(b) $0 = 24 - 1.25\,t$, so $t = 24 \div 1.25$
Answer: (a) 230.4 m (b) 19.2 s

2 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = \ln(2x - 1) + 3 - x$.
Find the equation of the normal to C at the point where $x = 1$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{2}{2x - 1} - 1$

At $x = 1$: $y = 2$ and $\frac{dy}{dx} = 1$

Normal gradient $= -1 \div 1 = -1$
$y - 2 = -1(x - 1)$, so $y = -x + 3$
Answer: $y = -x + 3$

3 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should $y = 6 + 1.5x$ not be used for $x = 14$?

(a) The points lie close to a straight line rising: strong positive correlation
(b) The data cover $x = 1$ to 10; $x = 14$ is outside this range
so using the line would be extrapolation and may be unreliable
Answer: (a) strong positive correlation (b) extrapolation outside the range of the data

4 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has 3rd term 19 and 5th term 23.
(a) Find the first term and the common difference.
(b) Find the sum of the first 30 terms.

$a + 2d = 19$ and $a + 4d = 23$
Subtract: $2d = 4$, so $d = 2$ and $a = 15$

$S_{30} = \frac{30}{2}(2 \times 15 + 29 \times 2) = 1320$

Answer: (a) $a = 15$, $d = 2$ (b) 1320

5 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 5 kg is pulled across smooth horizontal ice by a rope with tension 39 N, inclined at α above the horizontal, where $\tan\alpha = \frac{12}{5}$.

Take g = 9.8 m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 5/13$, $\sin\alpha = 12/13$
(a) Horizontal: $39 \times 5/13 = 5\,a$
(b) Vertical: $R + 39 \times 12/13 = 5 \times 9.8$
Answer: (a) 3 m s^{-2} (b) 13 N

Day 278 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = 2x - 2$. The point P is $(4, -4)$.
(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l = 2, so the perpendicular has gradient $-\frac{1}{2}$

$y + 4 = -\frac{1}{2}(x - 4)$, giving $x + 2y + 4 = 0$

(b) Solve $y = 2x - 2$ with $x + 2y + 4 = 0$: F = $(0, -2)$

$PF^2 = 4^2 + (-2)^2 = 20$

Answer: (a) $x + 2y + 4 = 0$ (b) F$(0, -2)$, PF $= 2\sqrt{5}$

7 Binomial: smallest n (All boards)

Given that each trap catches a rabbit independently with probability 0.2, find the least number of traps needed for the probability of at least one success to exceed 0.95.

P(at least one) $= 1 - 0.8^n > 0.95$

$0.8^n < 0.05$

$n > \frac{\log 0.05}{\log 0.8} = 13.43$

Answer: n = 14

8 Inequalities with exponents and logs (All boards)

Solve $3^x < 396$, giving your answer to 3 s.f.

Take logs: $x \ln 3 < \ln 396$

$x < \frac{\ln 396}{\ln 3} = 5.445$

Answer: $x < 5.44$

9 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $4x + 3y = k$ is a tangent to the circle
$(x - 3)^2 + (y + 2)^2 = 25$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (4, 3).
A unit step in this direction is (4, 3)/5, so the points of contact are
$(3, -2) \pm 5 \times (4, 3)/5 = (3, -2) \pm (4, 3)$
T = (7, 1) or $(-1, -5)$, and $k = 4x + 3y$ gives:
k = 31 or k = −19

Answer: k = 31 or k = −19

10 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The shaded region is the segment cut off by the chord AB of a circle with centre O and radius 10 cm. Angle AOB = 0.8 radians.
Find the area of the segment, to 3 s.f.

Sector area $= \frac{1}{2} \times 10^2 \times 0.8 = 40$

Triangle area $= \frac{1}{2} \times 10^2 \times \sin 0.8 = 35.87$

Segment = sector − triangle = 4.132 cm^2

Answer: 4.13 cm^2