Day 277 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^{\ln 3} \frac{e^x}{e^x + 1}\,dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{e^x}{e^x + 1}\,dx = \ln(e^x + 1)$

$= [\ln(e^x + 1)]_0^{\ln 3}$

$= \ln 2$

Answer: ln 2

2 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the times, t minutes, taken by 7 people to complete a puzzle gives $\Sigma t = 186$ and $\Sigma t^2 = 5456$.
Find the mean and the standard deviation (3 s.f.).

Mean = $186 \div 7 = 26.6$

Variance = $\frac{5456}{7} - (26.571)^2 = 73.39$

Standard deviation = $\sqrt{73.39} = 8.57$

Answer: mean 26.6, standard deviation 8.57

3 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Values of y = f(x) are shown.
x: 0, 1, 2, 3, 4
y: 2.1, 2.6, 2.1, 1.8, 1.6

Estimate $\int_0^4 f(x)\,dx$ using the trapezium rule.

$h = 1$

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{1}{2}[2.1 + 1.6 + 2(2.6 + 2.1 + 1.8)]$

$= 8.35$

Answer: 8.35

4 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg is pulled along rough horizontal ground by a horizontal force of 24.7 N. It accelerates at 2 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the coefficient of friction between the block and the ground.

$R = mg = 5 \times 9.8 = 49$
Resultant: $24.7 - F = 5 \times 2$, so $F = 14.7$
$F = \mu R$: $\mu = 14.7 \div 49$

Answer: 0.3

5 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre (−3, −2) and passes through the point (5, −4).
Find an equation of the circle.

$r^2 = (5 - (-3))^2 + (-4 - (-2))^2$
$= 64 + 4 = 68$

Answer: $(x + 3)^2 + (y + 2)^2 = 68$

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6 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves so that its position vector relative to O at time t s is $\mathbf{r} = (2t^2 - t)\mathbf{i} + (t^3 + t)\mathbf{j}$ metres.
Find the velocity when t = 1, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (4t - 1)\mathbf{i} + (3t^2 + 1)\mathbf{j}$
$t = 1$: $\mathbf{v} = (3)\mathbf{i} + (4)\mathbf{j}$
Speed $= \sqrt{(3^2 + 4^2)}$
Answer: $\mathbf{v} = (3)\mathbf{i} + (4)\mathbf{j}$ m s^{-1}; speed 5 m s^{-1}

7 Binomial: smallest n (All boards)

Given that each attempt at a task succeeds independently with probability 0.2, find the least number of attempts needed for the probability of at least one success to exceed 0.95.

$P(\text{at least one}) = 1 - 0.8^n > 0.95$
$0.8^n < 0.05$
$n > \dfrac{\log 0.05}{\log 0.8} = 13.43$
Answer: n = 14

8 Fixed-point iteration (All boards)

(a) Show that $x^2 - 4x - 2 = 0$ can be written as $x = \sqrt{4x + 2}$.
(b) Use $x_{n+1} = \sqrt{4x_n + 2}$ with $x_0 = 3$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^2 = 4x + 2$, so $x = \sqrt{4x + 2}$
(b) $x_1 = 3.7417$
$x_2 = 4.1191$
$x_3 = 4.2984$
Answer: $x_1 = 3.7417$, $x_2 = 4.1191$, $x_3 = 4.2984$

9 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\dfrac{8}{\sqrt{2}} - \sqrt{50}$, giving your answer in the form $k\sqrt{2}$.

$\dfrac{8}{\sqrt{2}} = \dfrac{8\sqrt{2}}{2} = 4\sqrt{2}$
$\sqrt{50} = 5\sqrt{2}$
$4\sqrt{2} - 5\sqrt{2} = -\sqrt{2}$
Answer: $-\sqrt{2}$

10 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of
$(2 + 4x)(1 + 3x)^6$.

$(1 + 3x)^6$ has x^2 coefficient 135 and x^3 coefficient 540
x^3 term of the product $= 2 \times 540 + 4 \times 135$
$= 1620$
Answer: 1620