Day 276 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 3\operatorname{cosec} 4x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{16}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = -12 \cot 4x \operatorname{cosec} 4x$

At $x = \frac{\pi}{16}$: $\frac{dy}{dx} = -12\sqrt{2}$

Answer: $\frac{dy}{dx} = -12 \cot 4x \operatorname{cosec} 4x$; gradient $-12\sqrt{2}$

2 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=5}^{12}(5r - 4)$.

There are $12 - 5 + 1 = 8$ terms

First term = 21, last term = 56

$S = \frac{8}{2}(21 + 56) = 308$

Answer: 308

3 Trig equations in radians (All boards)

Solve $\cos 2x = 0.8$ for $0 \leq x \leq \pi$,

giving answers to 3 significant figures.

Let $\theta = 2x$, so $\cos \theta = 0.8$ for $0 \leq \theta \leq 2\pi$

Calculator: $\theta = 0.6435$, then use the symmetry of the graph

$\theta = 0.6435, 5.640$

$x = \frac{\theta}{2}$

$x = 0.3218, 2.820$

Answer: x = 0.322, 2.82

4 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = k u_n + 1$, $u_1 = 2$.

Given that $u_3 = 4$, find the possible values of k.

$u_2 = 2k + 1$

$u_3 = k(2k + 1) + 1 = 2k^2 + k + 1$

$2k^2 + k + 1 = 4$, so $2k^2 + k - 3 = 0$

$(k - 1)(2k + 3) = 0$

$k = -\frac{3}{2}$ or $k = 1$

Answer: $k = -\frac{3}{2}$ or $k = 1$

5 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $(6\mathbf{i} - 3\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-2}.

Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(6\mathbf{i} - 3\mathbf{j}) + \mathbf{P} = 3(3\mathbf{i} + 4\mathbf{j}) = (9\mathbf{i} + 12\mathbf{j})$

$\mathbf{P} = 3\mathbf{i} + 15\mathbf{j}$

$|\mathbf{P}| = \sqrt{(3^2 + 15^2)}$

Answer: $\mathbf{P} = (3\mathbf{i} + 15\mathbf{j})$ N; $|\mathbf{P}| = 15.3$ N

6 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf $f(x) = kx^2$ for $0 \le x \le 6$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 3). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^3/3]$ from 0 to 6 $= k \times \frac{6^3}{3} = 1$, so $k = \frac{1}{72}$

(b) $P(X > 3) = \int_3^6 \frac{1}{72} x^2\,dx = 1 - \frac{3^3}{6^3} = \frac{7}{8}$

(c) $E(X) = \int_0^6 x\,f(x)\,dx = \frac{1}{72} \times \frac{6^4}{4} = \frac{9}{2}$

Answer: (a) $k = \frac{1}{72}$ (b) $\frac{7}{8}$ (c) $\frac{9}{2}$

7 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = (5 + y)e^{-x}$,
given that y = 1 when x = 0.

$\frac{1}{5+y}\,dy = e^{-x}\,dx$, so $\ln(5 + y) = -e^{-x} + c$
x = 0: $\ln 6 = -1 + c$, so $c = 1 + \ln 6$
$5 + y = 6\,e^{1 - e^{-x}}$, so $y = 6\,e^{1 - e^{-x}} - 5$
Answer: $y = 6e^{1 - e^{-x}} - 5$

8 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the times, t minutes, taken by 10 people to complete a puzzle gives $\Sigma t = 275$ and $\Sigma t^2 = 8579$.
Find the mean and the standard deviation (3 s.f.).

Mean $= 275 \div 10 = 27.5$
Variance $= \frac{8579}{10} - (27.500)^2 = 101.7$
Standard deviation $= \sqrt{101.7} = 10.1$
Answer: mean 27.5, standard deviation 10.1

9 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = t^2$, $y = 3t$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve.

From $y = 3t$: $t = \frac{y}{3}$
$x = (\frac{y}{3})^2 = \frac{y^2}{9}$
So $y^2 = 9x$
Answer: $y^2 = 9x$

10 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 5 kg rests on a rough horizontal floor, with coefficient of friction 0.5. A horizontal force of 20 N is applied to the box.
Take $g = 9.8$ m s^{-2}.
Determine whether the box moves. Find the friction force, or the acceleration if it moves.

R = 5 × 9.8 = 49, maximum friction $\mu R = 0.5 \times 49 = 24.5$
20 < 24.5 so the box stays at rest
Friction balances the applied force: F = 20
Answer: Does not move; friction = 20 N