Day 275 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic series is 9 and the common difference is 6.
The sum of the first n terms is 1080. Find the value of n.

$S_n = \frac{n}{2}(2 \times 9 + (n - 1) \times 6) = 1080$

$6n^2 + 12n - 2160 = 0$
$6(n - 18)(n + 20) = 0$
n must be positive, so n = 18
Answer: n = 18

2 Chain rule (All boards)

Given that $y = \ln(4x^2 + 3)$,
find $\frac{dy}{dx}$, and the gradient of the curve when x = 2.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function
$\frac{dy}{dx} = \frac{8x}{4x^2 + 3}$

When x = 2: gradient = $\frac{16}{19}$

Answer: $\frac{dy}{dx} = \frac{8x}{4x^2 + 3}$; gradient $\frac{16}{19}$

3 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(-3\mathbf{i} + 8\mathbf{j})$ N and $(-4\mathbf{j})$ N act on a particle of mass 4 kg.
Find the magnitude of the acceleration of the particle.

Resultant = $(-3\mathbf{i} + 8\mathbf{j}) + (-4\mathbf{j}) = -3\mathbf{i} + 4\mathbf{j}$
$|R| = \sqrt{(-3^2 + 4^2)} = 5$ N
a = F/m = 5 ÷ 4
Answer: 1.25 m s^{-2}

4 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 12 cm and area $\frac{144\pi}{5}$ cm^2.

Find the angle of the sector in degrees.

$\frac{1}{2}r^2\theta = \frac{144\pi}{5}$, so $\theta = \frac{2 \times \frac{144\pi}{5}}{12^2} = \frac{2\pi}{5}$ radians

$\theta = \frac{2\pi}{5} \times \frac{180}{\pi} = 72°$

Answer: 72°

5 Binomial mean and variance (All boards)

X has a binomial distribution with mean 1.6 and variance 1.28.
(a) Find n and p.
(b) Find P(X = 1) to 3 significant figures.

(a) np = 1.6 and np(1 − p) = 1.28
1 − p = 1.28 ÷ 1.6 = 0.8, so p = 0.2 and n = 1.6 ÷ 0.2 = 8
(b) $P(X = 1) = {}^8C_1 \times 0.2^1 \times 0.8^7 = 0.336$
Answer: (a) n = 8, p = 0.2 (b) 0.336

6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $7x - y = 35$ and line l_2 has equation $7x - 12y = -42$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (6, 7)
l_1 meets y = 0 at x = 5, and l_2 meets y = 0 at $x = -6$
Base = 11, height = 7
Area = $\frac{1}{2}$ × base × height = $\frac{77}{2}$

Answer: $\frac{77}{2}$

7 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $3 \sin x + 5 \cos^2 x - 5 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $3 \sin x - 5 \sin^2 x = 0$, i.e. $5 \sin^2 x - 3 \sin x = 0$
$(5 \sin x - 3)(\sin x) = 0$, so $\sin x = \frac{3}{5}$ or 0
$\sin x = \frac{3}{5}$: x = 36.9°, 143.1°
$\sin x = 0$: x = 0.0°, 180.0°, 360.0°
Answer: x = 0.0°, 36.9°, 143.1°, 180.0°, 360.0°

8 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the marks in a mock exam for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Year 12: 50 − 20 = 30. Year 13: 35 − 20 = 15
(b) Medians: 35 (Year 12) and 25 (Year 13), so Year 12 has the higher median
IQRs: Year 12 has the larger IQR, so its scores are more spread out
Answer: (a) 30 and 15 (b) Year 12 has the higher median; Year 12 is more spread out (larger IQR), Year 13 more consistent

9 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a cube number cannot be written in the form $7k + 2$, where k is an integer.

Write the integer as $n = 7q + j$, where $j = 0, 1, ..., 6$.
n^3 leaves the same remainder as j^3 when divided by 7.
$j = 0, 1, 2, 3, 4, 5, 6$ give remainders 0, 1, 1, 6, 1, 6, 6.
The only possible remainders are 0, 1, 6, never 2.
Answer: possible remainders 0, 1, 6, never 2

10 Work-energy: sliding to rest (CIE)

A block of mass 3 kg is projected along a rough horizontal surface with speed 8.4 m s^{-1} and comes to rest after sliding 12 m.
Take g = 9.8 m s^{-2}.
Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE
$\mu \times 3 \times 9.8 \times 12 = ½ \times 3 \times 8.4^2$
$352.8\,\mu = 105.84$
Answer: 0.3